LESSON

1.1

Building Blocks of Geometry

Nature's Great Book is written in mathematical symbols.

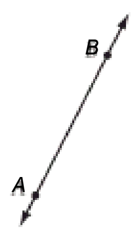
GALILEO GALILEI

Three building blocks of geometry are points, lines, and planes. A **point** is the most basic building block of geometry. It has no size. It has only location. You represent a point with a dot, and you name it with a capital letter. The point shown below is called P.

Mathematical model of a point

A tiny seed is a physical model of a point. A point, however, is smaller than any seed that ever existed.

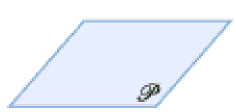
A **line** is a straight, continuous arrangement of infinitely many points. It has infinite length, but no thickness. It extends forever in two directions. You name a line by giving the letter names of any two points on the line and by placing the line symbol above the letters, for example, $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$.



Mathematical model of a line

A piece of spaghetti is a physical model of a line. A line, however, is longer, straighter, and thinner than any piece of spaghetti ever made.

A **plane** has length and width, but no thickness. It is like a flat surface that extends infinitely along its length and width. You represent a plane with a four-sided figure, like a tilted piece of paper, drawn in perspective. Of course, this actually illustrates only part of a plane. You name a plane with a script capital letter, such as $\mathcal{P}$.



Mathematical model of a plane

A flat piece of rolled-out dough is a physical model of a plane. A plane, however, is broader, wider, and thinner than any piece of dough you could ever roll.

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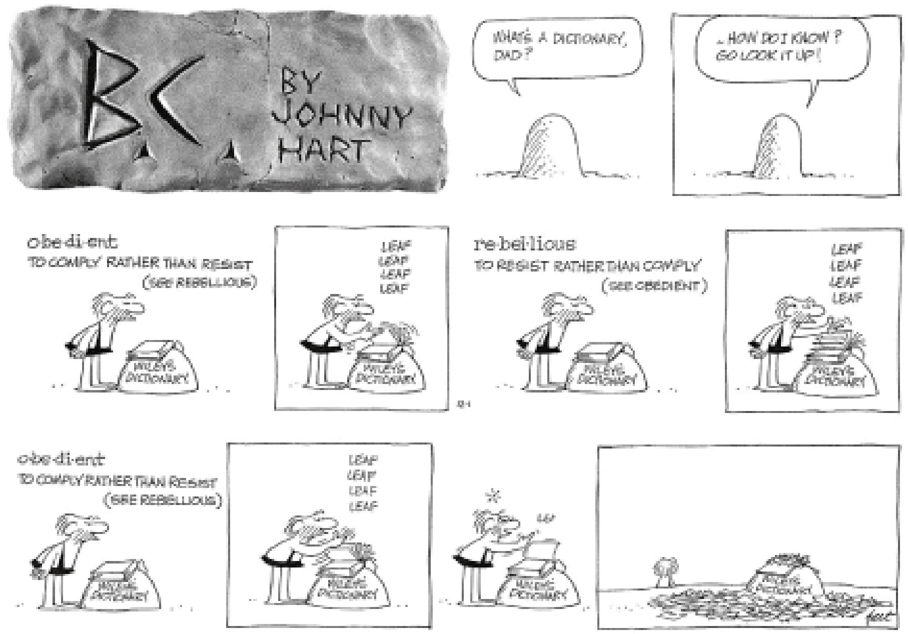
It can be difficult to explain what points, lines, and planes are even though you may recognize them. Early mathematicians tried to define these terms.



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The ancient Greeks said, "A point is that which has no part. A line is breadthless length." The Mohist philosophers of ancient China said, "The line is divided into parts, and that part which has no remaining part is a point." Those definitions don't help much, do they?

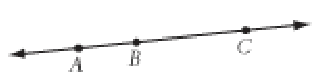
A **definition** is a statement that clarifies or explains the meaning of a word or a phrase. However, it is impossible to define point, line, and plane without using words or phrases that themselves need definition. So these terms remain undefined. Yet, they are the basis for all of geometry.

Using the undefined terms *point, line,* and *plane,* you can define all other geometry terms and geometric figures. Many are defined in this book, and others will be defined by you and your classmates.

Keep a definition list in your notebook, and each time you encounter new geometry vocabulary, add the term to your list. Illustrate each definition with a simple sketch.

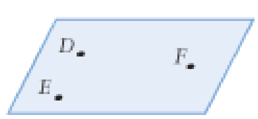
Here are your first definitions. Begin your list and draw sketches for all definitions.

Collinear means on the same line.



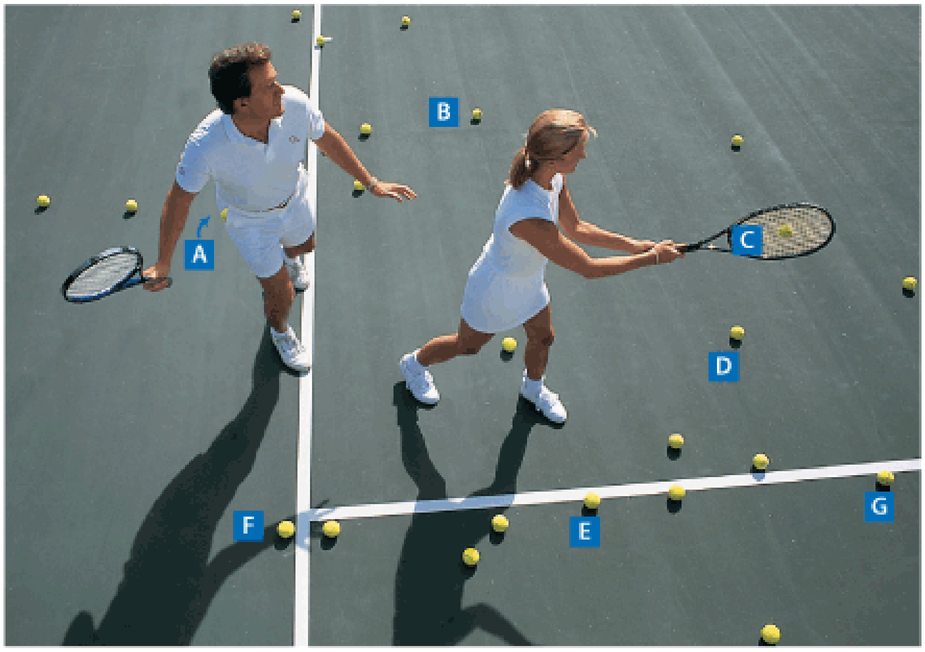
Points *A, B,* and *C* are collinear.

Coplanar means on the same plane.

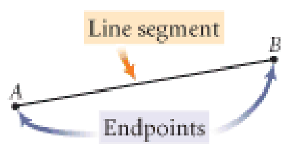


Points *D, E,* and *F* are coplanar.

Ball A is in the pocket of the man. Ball C is on the woman's racquet. All other balls are on the tennis court. Name three balls that are collinear. Name three balls that are coplanar but not collinear. Name four balls that are not coplanar.



A **line segment** consists of two points called the **endpoints** of the segment and all the points between them that are collinear with the two points.

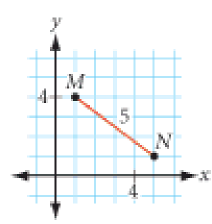


You can write line segment AB, using a segment symbol, as $\overline{AB}$ or $\overline{BA}$. There are two ways to write the length of a segment. You can write AB = 2 in., meaning the distance from A to B is 2 inches. You can also use an m for "measure" in front of the segment name, and write the distance as $m\overline{AB} = 2$ in. If no measurement units are used for the length of a segment, it is understood that the choice of units is not important or is based on the length of the smallest square in the grid.

Figure A

$AB = 2$ in., or $m\overline{AB} = 2$ in.

Figure B



$MN = 5$ units, or $m\overline{MN} = 5$ units

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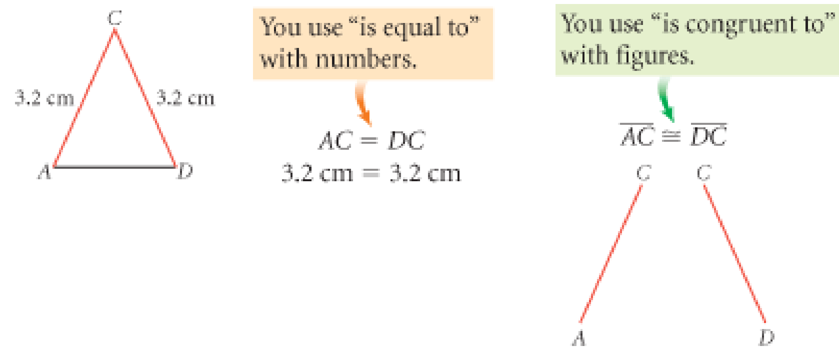
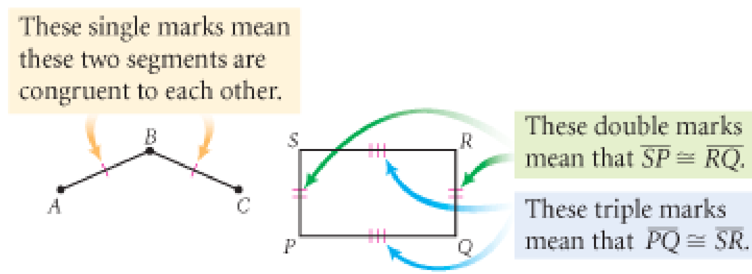
Two segments are **congruent** if and only if they have equal measures, or lengths.



When drawing figures, you show congruent segments by making identical markings.



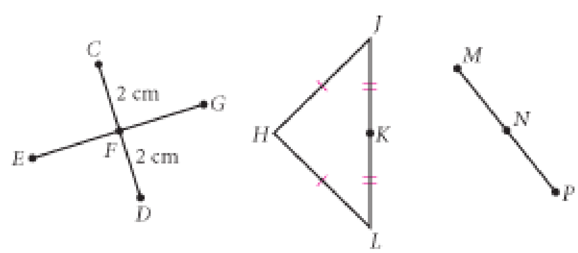
The **midpoint** of a segment is the point on the segment that is the same distance from both endpoints. The midpoint **bisects** the segment, or divides the segment into two congruent segments.

EXAMPLE

Study the diagrams below.

a. Name each midpoint and the segment it bisects.

b. Name all the congruent segments. Use the congruence symbol to write your answers.



Solution

Look carefully at the markings and apply the midpoint definition.

a. $CF = FD$, so F is the midpoint of $\overline{CD}$; $\overline{JK} \cong \overline{KL}$, so K is the midpoint of $\overline{JL}$.

b. $\overline{CF} \cong \overline{FD}$, $\overline{HJ} \cong \overline{HL}$, and $\overline{JK} \cong \overline{KL}$.

Even though $\overline{EF}$ and $\overline{FG}$ appear to have the same length, you cannot assume they are congruent without the markings. The same is true for $\overline{MN}$ and $\overline{NP}$.

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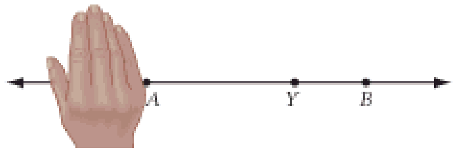
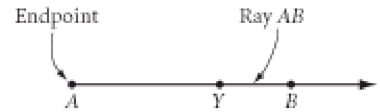
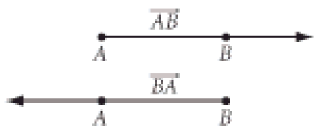
Ray AB is the part of $\overleftrightarrow{AB}$ that contains point A and all the points on $\overleftrightarrow{AB}$ that are on the same side of point A as point B. Imagine cutting off all the points to the left of point A.

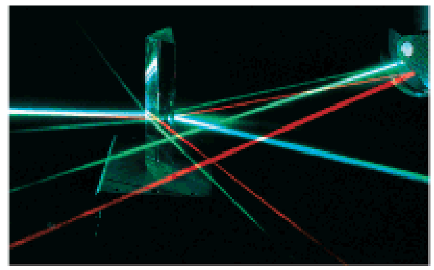




In the figure above, $\overrightarrow{AY}$ and $\overrightarrow{AB}$ are two ways to name the same ray. Note that $\overrightarrow{AB}$ is not the same as $\overrightarrow{BA}$!

A ray begins at a point and extends infinitely in one direction. You need two letters to name a ray. The first letter is the endpoint of the ray, and the second letter is any other point that the ray passes through.





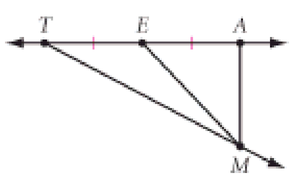
Physical model of a ray: beams of light

Investigation
Mathematical Models

In this lesson, you encountered many new geometry terms. In this investigation you will work as a group to identify models from the real world that represent these terms and to identify how they are represented in diagrams.

Step 1 Look around your classroom and identify examples of each of these terms: point, line, plane, line segment, congruent segments, midpoint of a segment, and ray.

Step 2 Identify examples of these terms in the photograph at right.



Step 3 Identify examples of these terms in the figure above.

Step 4 Explain in your own words what each of these terms means.

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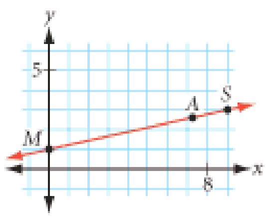
EXERCISES

1. In the photos below identify the physical models that represent a point, segment, plane, collinear points, and coplanar points.

For Exercises 2–4, name each line in two different ways.

2. 

3. 

4. 

For Exercises 5–7, draw two points and label them. Then use a ruler to draw each line. Don't forget to use arrowheads to show that the line extends indefinitely.

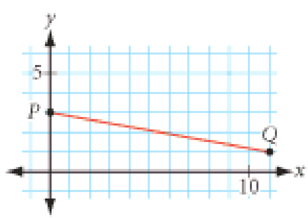
5. $\overleftrightarrow{AB}$

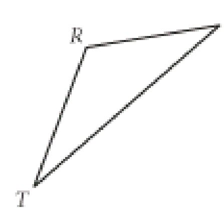
6. $\overleftrightarrow{KL}$

7. $\overleftrightarrow{DE}$ with $D(-3, 0)$ and $E\,(0, -3)$

For Exercises 8–10, name each line segment.

8. 

9. 

10. 

For Exercises 11 and 12, draw and label each line segment.

11. $\overline{AB}$

12. $\overline{RS}$ with $R(0, 3)$ and $S(-2, 11)$

For Exercises 13 and 14, use your ruler to find the length of each line segment to the nearest tenth of a centimeter. Write your answer in the form $m\overline{AB} = \underline{\ ?\ }$.

13. 

14. 

For Exercises 15–17, use your ruler to draw each segment as accurately as you can. Label each segment.

15. $AB = 4.5$ cm

16. $CD = 3$ in.

17. $EF = 24.8$ cm

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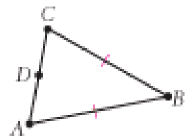
18. Name each midpoint and the segment it bisects.

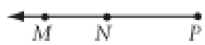


19. Draw two segments that have the same midpoint. Mark your drawing to show congruent segments.

20. Draw and mark a figure in which M is the midpoint of $\overline{ST}$, $SP = PT$, and T is the midpoint of $\overline{PQ}$.

For Exercises 21–23, name the ray in two different ways.

21. 

22. 

23. 

For Exercises 24–26, draw and label each ray.

24. $\overrightarrow{AB}$

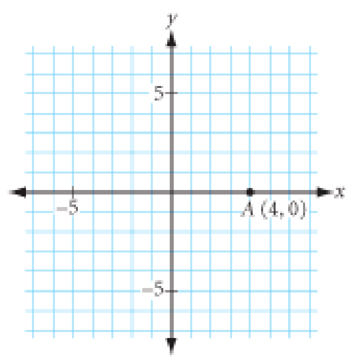
25. $\overrightarrow{YX}$

26. $\overrightarrow{MN}$

27. Draw a plane containing four coplanar points A, B, C, and D, with exactly three collinear points A, B, and D.

28. Given two points A and B, there is only one segment that you can name: $\overline{AB}$. With three collinear points A, B, and C, there are three different segments that you can name: $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. With five collinear points A, B, C, D, and E, how many different segments can you name?

For Exercises 29–31, draw axes on graph paper and locate point A (4, 0) as shown.



29. Draw $\overline{AB}$ where point B has coordinates (2, −6).

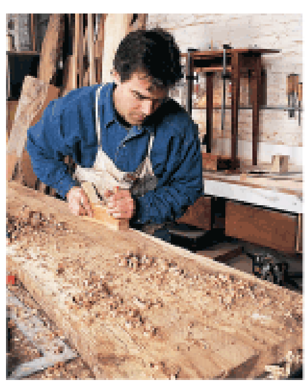
30. Draw $\overrightarrow{OM}$ with endpoint (0, 0) that goes through point M (2, 2).

31. Draw $\overleftrightarrow{CD}$ through points C (−2, 1) and D (−2, −3).

Career CONNECTION

Woodworkers use a tool called a plane to shave a rough wooden surface to create a perfectly smooth planar surface. The smooth board can then be made into a tabletop, a door, or a cabinet.

Woodworking is a very precise process. Producing high-quality pieces requires an understanding of lines, planes, and angles as well as careful measurements.



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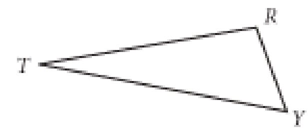
32. If the signs of the coordinates of collinear points $P(-6, -2)$, $Q(-5, 2)$, and $R(-4, 6)$ are reversed, are the three new points still collinear? Draw a picture and explain why.

33. Draw a segment with midpoint $N(-3, 2)$. Label it $\overline{PQ}$.

34. Copy triangle TRY shown at right. Use your ruler to find the midpoint A of side $\overline{TR}$ and the midpoint G of side $\overline{TY}$. Draw $\overline{AG}$.



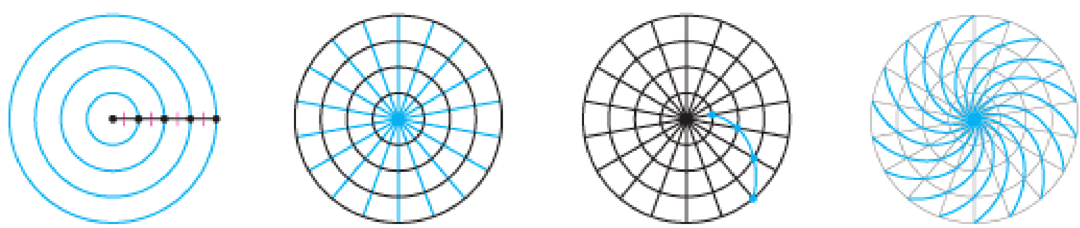
35. Use your ruler to draw a triangle with side lengths 8 cm and 11 cm. Explain your method. Can you draw a second triangle with these two side lengths that looks different from the first?

project

SPIRAL DESIGNS

The circle design shown below is used in a variety of cultures to create mosaic decorations. The spiral design may have been inspired by patterns in nature. Notice that the seeds on the sunflower also spiral out from the center.

Create and decorate your own spiral design. Here are the steps to make the spirals. The more circles and radii you draw, the more detailed your design will be.



Step 1 Step 2 Step 3 Step 4

- A completed spiral.
- Coloring or decorations that make the spiral stand out.

For help, see the **Dynamic Geometry Exploration** Spiral Designs at www.keymath.com/DG. keymath.com/DG

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ALGE • USING YOUR ALGEBRA SKILLS 1 • USING

Midpoint

A midpoint is the point on a line segment that is the same distance from both endpoints.

You can think of a midpoint as being halfway between two locations. You know how to mark a midpoint. But when the position and location matter, such as in navigation and geography, you can use a coordinate grid and some algebra to find the exact location of the midpoint. You can calculate the coordinates of the midpoint of a segment on a coordinate grid using a formula.

Coordinate Midpoint Property

If (x_1, y_1) and (x_2, y_2) are the coordinates of the endpoints of a segment, then the coordinates of the midpoint are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

History CONNECTION

Surveyors and mapmakers of ancient Egypt, China, Greece, and Rome used various coordinate systems to locate points. Egyptians made extensive use of square grids and used the first known rectangular coordinates at Saqqara around 2650 B.C.E. By the 17th century, the age of European exploration, the need for accurate maps and the development of easy-to-use algebraic symbols gave rise to modern coordinate geometry. Notice the lines of latitude and longitude in this 17th-century map.

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EXAMPLE

Segment AB has endpoints (−8, 5) and (3, −6). Find the coordinates of the midpoint of $\overline{AB}$.

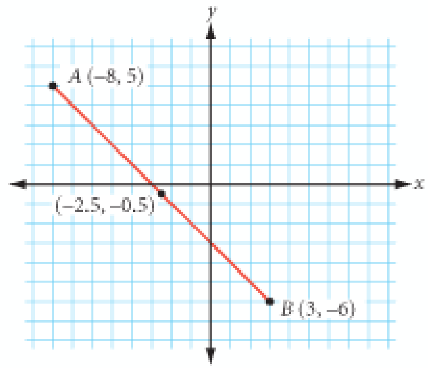
Solution

The midpoint is not on a grid intersection point, so we can use the coordinate midpoint property.

$$= \frac{x_1 + x_2}{2} = \frac{-8 + 3}{2} = -2.5$$

$$= \frac{y_1 + y_2}{2} = \frac{5 + (-6)}{2} = -0.5$$

The midpoint of $\overline{AB}$ is (−2.5, −0.5).



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EXERCISES

For Exercises 1–3, find the coordinates of the midpoint of the segment with each pair of endpoints.

1. (12, −7) and (−6, 15) **2.** (−17, −8) and (−1, 11) **3.** (14, −7) and (−3, 18)

4. One endpoint of a segment is (12, −8). The midpoint is (3, 18). Find the coordinates of the other endpoint.

5. A classmate tells you, "Finding the coordinates of a midpoint is easy. You just find the averages." Is there any truth to it? Explain what you think your classmate means.

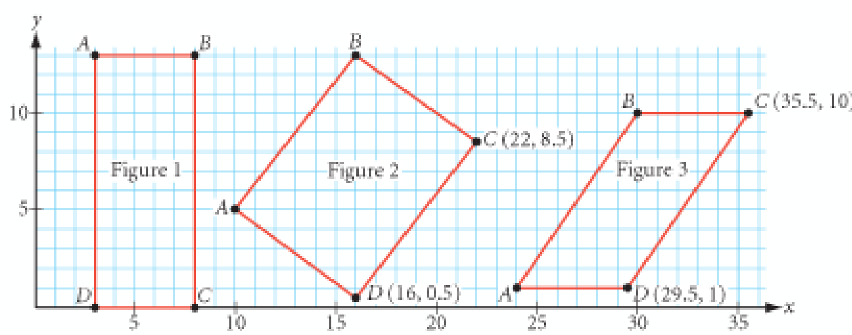
6. Find the two points on $\overline{AB}$ that divide the segment into three congruent parts. Point A has coordinates (0, 0) and point B has coordinates (9, 6). Explain your method.

7. Describe a way to find points that divide a segment into fourths.

8. In each figure below, imagine drawing the diagonals $\overline{AC}$ and $\overline{BD}$.

a. Find the midpoint of $\overline{AC}$ and the midpoint of $\overline{BD}$ in each figure.

b. What do you notice about the midpoints?



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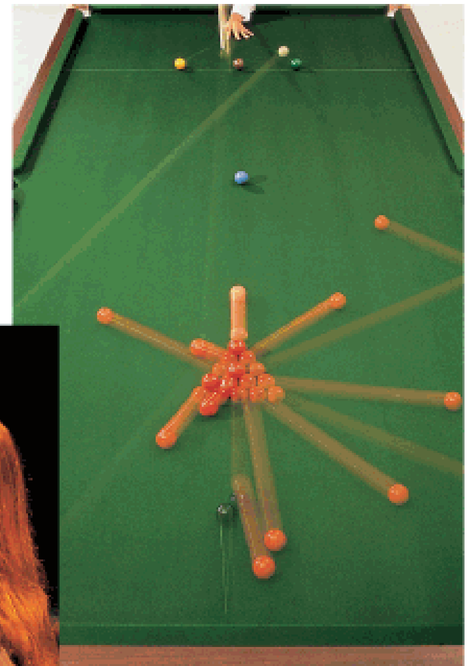
LESSON 1.2

Poolroom Math

Inspiration is needed in geometry, just as much as in poetry.
ALEKSANDR PUSHKIN

People use angles every day. Plumbers measure the angle between connecting pipes to make a good fitting. Woodworkers adjust their saw blades to cut wood at just the correct angle. Air traffic controllers use angles to direct planes. And good pool players must know their angles to plan their shots.



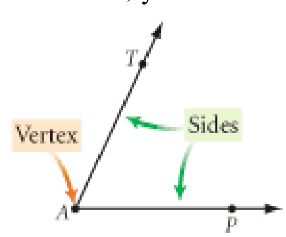
Is the angle between the two hands of the wristwatch smaller than the angle between the hands of the large clock?

"Little Benji," the wristwatch

Big Ben at the Houses of Parliament in London, England

You can use the terms that you defined in Lesson 1.1 to write a precise definition of angle. An **angle** is formed by two rays that share a common endpoint, provided that the two rays are noncollinear. In other words, the rays cannot lie on the same line. The common endpoint of the two rays is the **vertex** of the angle. The two rays are the **sides** of the angle.

You can name the angle in the figure below angle *TAP* or angle *PAT*, or use the angle symbol and write $\angle TAP$ or $\angle PAT$. Notice that the vertex must be the middle letter, and the first and last letters each name a point on a different ray. Since there are no other angles with vertex *A*, you can also simply call this $\angle A$



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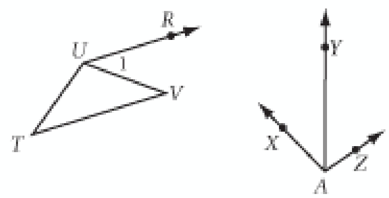
EXAMPLE A

Name all the angles in these drawings.



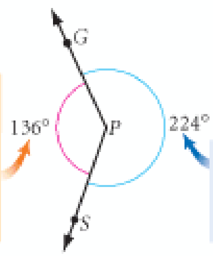
► Solution

The angles are $\angle T$, $\angle V$, $\angle TUV$, $\angle 1$, $\angle TUR$, $\angle XAY$, $\angle YAZ$, and $\angle XAZ$. (Did you get them all?) Notice that $\angle 1$ is a shorter way to name $\angle RUV$.

Which angles in Example A seem big to you? Which seem small?

The **measure of an angle** is the smallest amount of rotation about the vertex from one ray to the other, measured in **degrees.** According to this definition, the measure of an angle can be any value between 0° and 180°. The largest amount of rotation less than 360° between the two rays is called the **reflex measure of an angle.**

The measure of $\angle GPS$ is 136°, the smallest amount of rotation from $\overrightarrow{PG}$ to $\overrightarrow{PS}$.



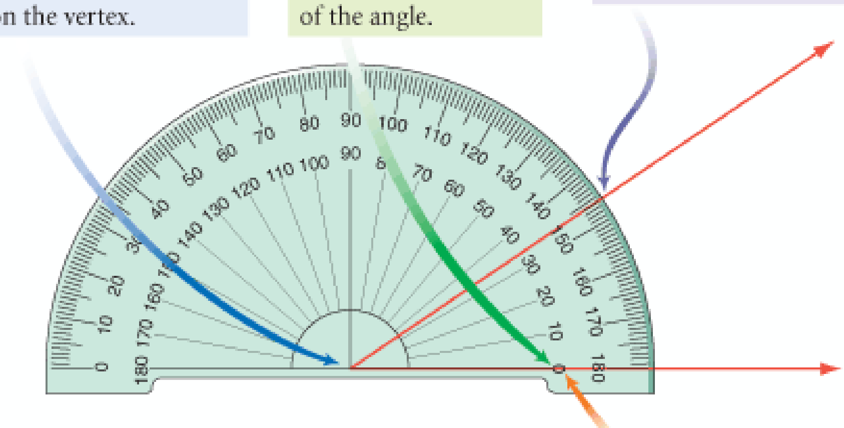
The reflex measure of $\angle GPS$ is 224°.

The geometry tool you use to measure an angle is a **protractor.** Here's how you use it.

Step 1: Place the center mark of the protractor on the vertex.

Step 2: Line up the 0-mark with one side of the angle.

Step 3: Read the measure on the protractor scale.



Step 4: Be sure you read the scale that has the 0-mark you are using! The angle in the diagram measures 34° and not 146°.

A television antenna is a physical model of an angle. Note that changing the length of the antenna doesn't change the angle.

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For a visual tutorial on how to use a protractor, see the **Dynamic Geometry Exploration** at Protractor www.keymath.com/DG .

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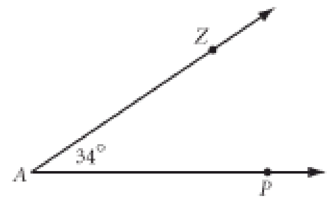
Career CONNECTION

In sports medicine, specialists may examine the healing rate of an injured joint by its angle of recovery. For example, a physician may assess how much physical therapy a patient needs by measuring the degree to which a patient can bend his or her ankle from the floor.

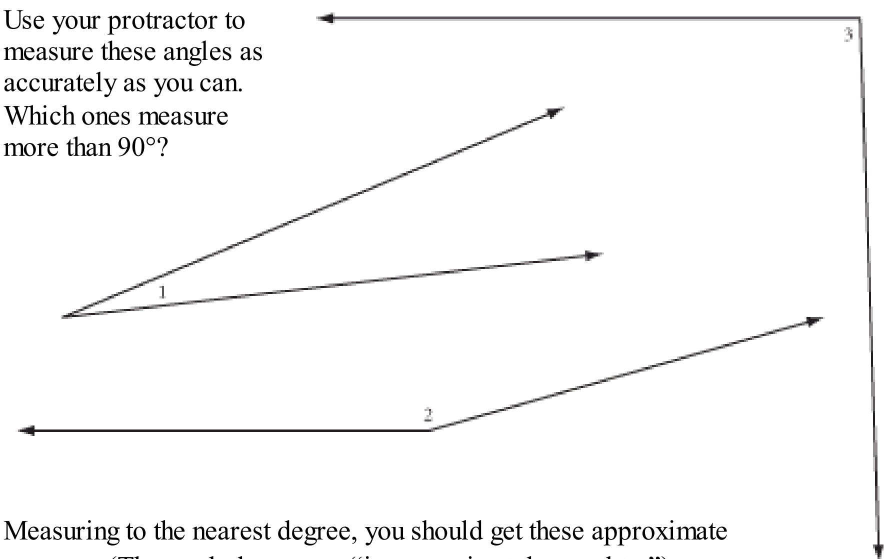
To show the measure of an angle, use an m before the angle symbol. For example, $m\angle ZAP = 34°$ means the measure of $\angle ZAP$ is 34 degrees.



$m\angle ZAP = 34°$

EXAMPLE B

Use your protractor to measure these angles as accurately as you can. Which ones measure more than 90°?



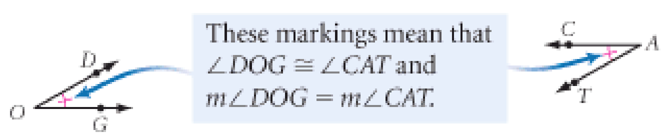
▶ Solution

Measuring to the nearest degree, you should get these approximate answers. (The symbol ≈ means "is approximately equal to.")

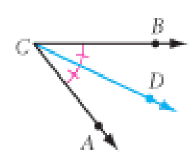
$m\angle 1 \approx 16°$ $\quad$ $m\angle 3 \approx 92°$

$m\angle 2 \approx 164°$ $\quad$ $\angle 2$ and $\angle 3$ measure more than 90°.

Two angles are **congruent** if and only if they have equal measure. You use identical markings to show that two angles in a figure are congruent.



A ray is the **angle bisector** if it contains the vertex and divides the angle into two congruent angles. In the figure at right, $\overrightarrow{CD}$ bisects $\angle ACB$ so that $\angle ACD \cong \angle BCD$.



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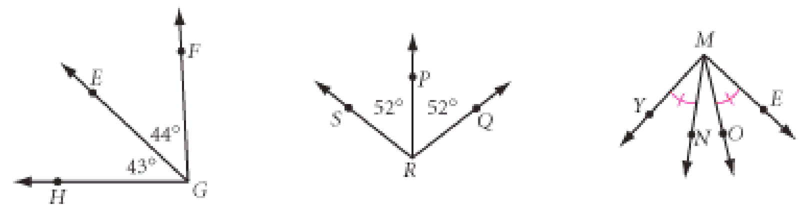
EXAMPLE C

Look for angle bisectors and congruent angles in the figures below.

a. Name each angle bisector and the angle it bisects.

b. Name all the congruent angles in the figure. Use the congruence symbol and name the angles so there is no confusion about which angle you mean.



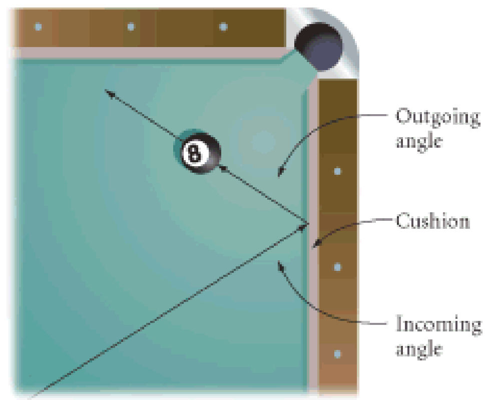
▶ Solution

a. Use the angle bisector definition. $\angle SRP \cong \angle PRQ$, so $\overrightarrow{RP}$ bisects $\angle SRQ$.
b. $\angle \boldsymbol{SRP} \cong \angle \boldsymbol{PRQ}$, $\angle \boldsymbol{YMN} \cong \angle \boldsymbol{OME}$, and $\angle \boldsymbol{YMO} \cong \angle \boldsymbol{EMN}$.

Investigation
Virtual Pool

You will need

- the worksheet Poolroom Math
- a protractor



Pocket billiards, or pool, is a game of angles. When a ball bounces off the pool table's cushion, its path forms two angles with the edge of the cushion. The **incoming angle** is formed by the cushion and the path of the ball approaching the cushion.

The **outgoing angle** is formed by the cushion and the path of the ball leaving the cushion. As it turns out, the measure of the outgoing angle equals the measure of the incoming angle.

Computer scientist Nesli O'Hare is also a professional pool player.

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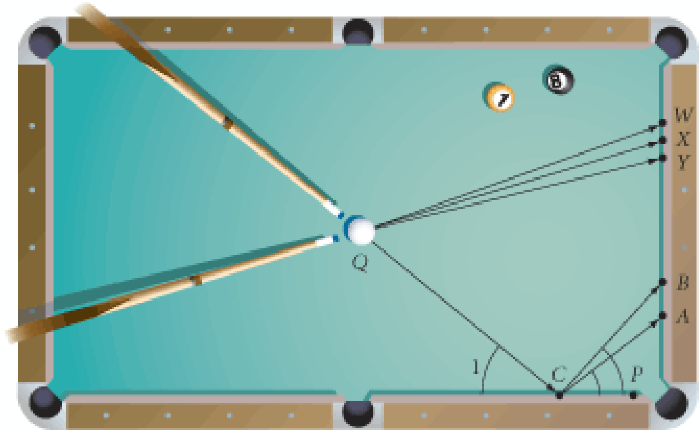
Use your protractor to study these shots.



Step 1 Use your protractor to find the measure of $\angle 1$. Which is the correct outgoing angle? Which point—*A* or *B*—will the ball hit?

Step 2 Which point on the cushion—*W, X,* or *Y*—should the white ball hit so that the ray of the outgoing angle passes through the center of the 8-ball?

Step 3 Compare your results with your group members' results. Does everyone agree?

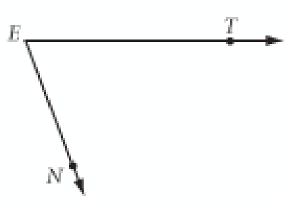
Step 4 How would you hit the white ball against the cushion so that the ball passes over the same spot on the way back?

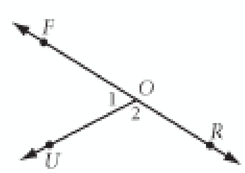
Step 5 How would you hit the ball so that it bounces off three different points on the cushions without ever touching cushion $\overleftrightarrow{CP}$?

EXERCISES

Keymath.com Practice Your Skills

1. Name each angle in three different ways.





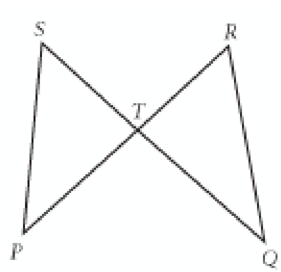
For Exercises 2–4, draw and label each angle.

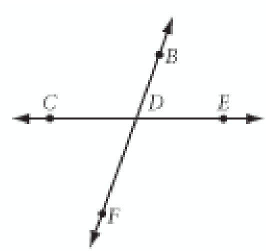
2. $\angle TAN$

3. $\angle BIG$

4. $\angle SML$

5. For each figure at right, list the angles that you can name using only the vertex letter.





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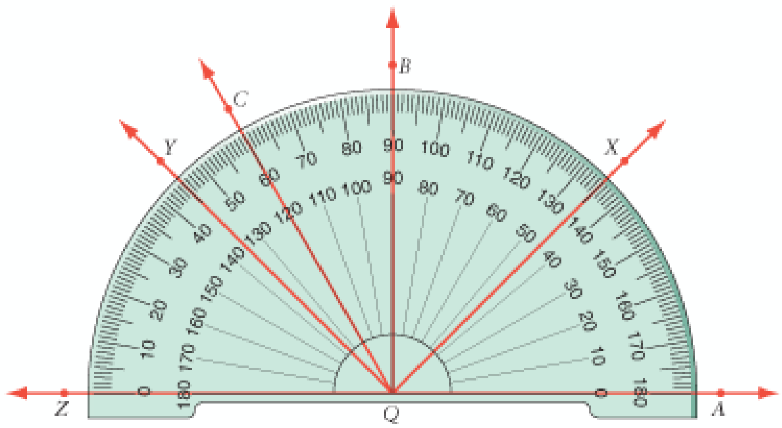
6. Draw a figure that contains at least three angles and requires three letters to name each angle.

For Exercises 7–14, find the measure of each angle to the nearest degree.



If an exercise has an ⓗ at the end, you can find a hint to help you in Hints for Selected Exercises at the back of the book.

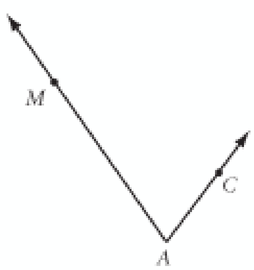
7. $m\angle AQB \approx$? **8.** $m\angle AQC \approx$? **9.** $m\angle XQA \approx$? **10.** $m\angle AQY \approx$?

11. $m\angle ZQY \approx$? **12.** $m\angle ZQX \approx$? **13.** $m\angle CQB \approx$? ⓗ **14.** $m\angle XQY \approx$?

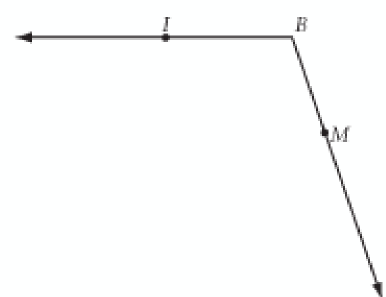
15. **Adjacent Angles** $\angle XQA$ and $\angle XQY$ share a vertex and a side. Taken together they form the larger angle $\angle AQY$. Compare their measures. Does $m\angle XQA + m\angle XQY = m\angle AQY$?

For Exercises 16–20, use your protractor to find the measure of the angle to the nearest degree.

16. $m\angle MAC \approx$?



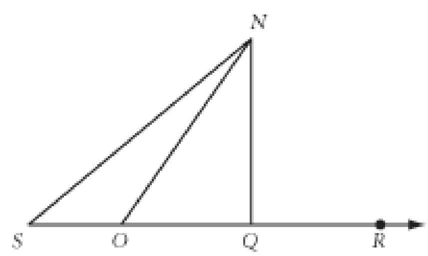
17. $m\angle IBM \approx$?



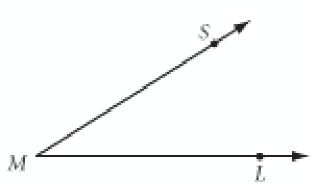
18. $m\angle S \approx$?

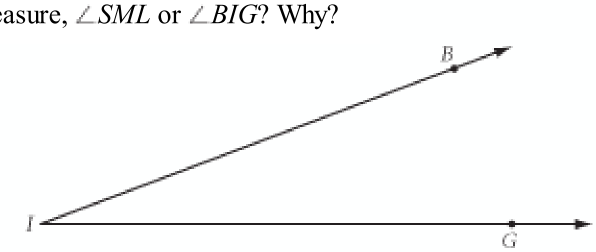
19. $m\angle SON \approx$?

20. $m\angle NOR \approx$?



21. Which angle below has the greater measure, $\angle SML$ or $\angle BIG$? Why?



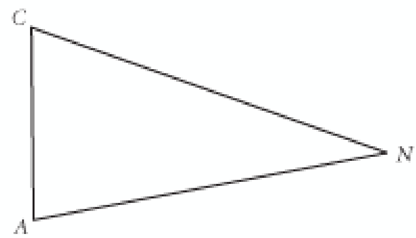


For Exercises 22–24, use your protractor to draw angles with these measures. Label them.

22. $m\angle A = 44°$ **23.** $m\angle B = 90°$ **24.** $m\angle CDE = 135°$

25. Use your protractor to draw the angle bisector of $\angle A$ in Exercise 22 and the angle bisector of $\angle D$ in Exercise 24. Use markings to show that the two halves are congruent.

26. Copy triangle *CAN* shown at right. Use your protractor to find the angle bisector of $\angle A$. Label the point where it crosses $\overline{CN}$ point *Y*. Use your ruler to find the midpoint of $\overline{CN}$ and label it *D*. Are *D* and *Y* the same point?



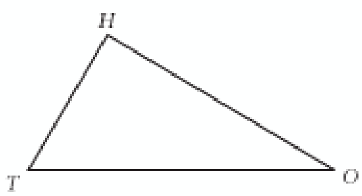
For Exercises 27–29, draw a clock face with hands to show these times.

27. 3:30 (h) **28.** 3:40 **29.** 3:15

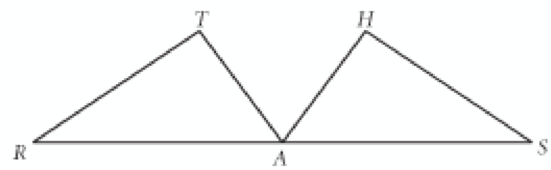
30. Give an example of a time when the angle made by the hands of the clock will be greater than 90°.

For Exercises 31–34, copy each figure and mark it with all the given information.

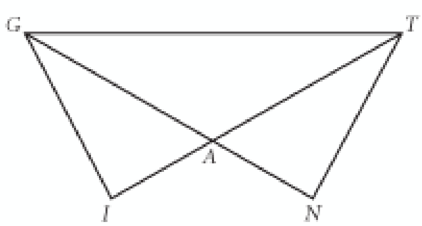
31. $TH = 6$
$m\angle THO = 90°$
$OH = 8$



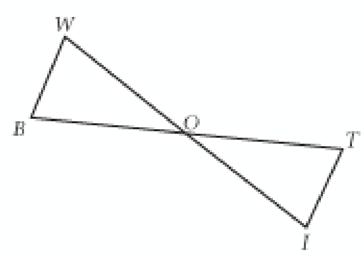
32. $RA = SA$
$m\angle T = m\angle H$
$RT = SH$



33. $AT = AG$ $\angle AGT \cong \angle ATG$
$AI = AN$ $GI = TN$

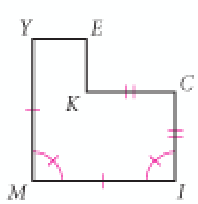


34. $\overline{BW} \cong \overline{TI}$ $\angle WBT \cong \angle ITB$
$\overline{WO} \cong \overline{IO}$ $\angle BWO \cong \angle TIO$

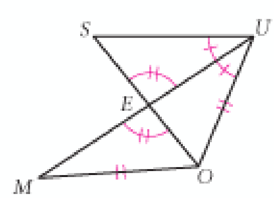


For Exercises 35 and 36, write down what you know from the markings. Do not use your protractor or your ruler.

35. $MI = \underline{\ ?\ }$
$IC = \underline{\ ?\ }$
$m\angle M = \underline{\ ?\ }$



36. $\angle MEO \cong \underline{\ ?\ }$
$\angle SUE \cong \underline{\ ?\ }$
$OU = \underline{\ ?\ }$



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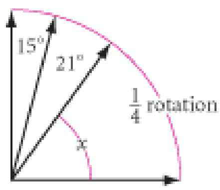
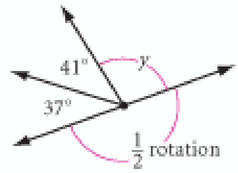
For Exercises 37–39, do not use a protractor. Recall from Chapter 0 that a complete rotation around a point is 360°. Find the angle measures represented by each letter.

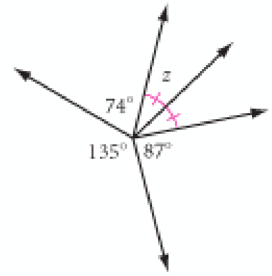
37.



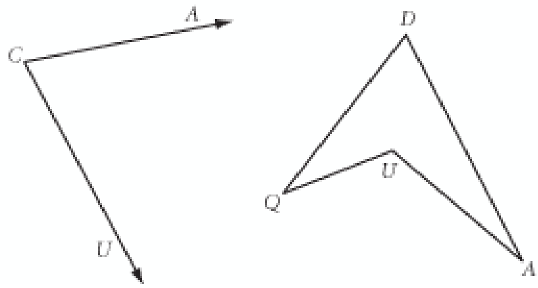
38.



39.



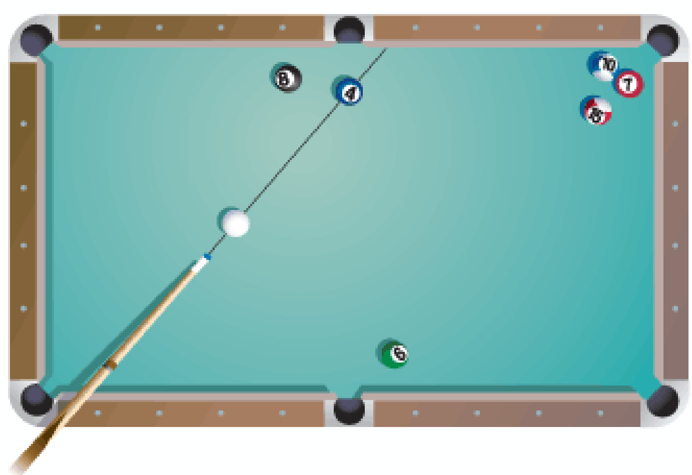
40. Use your protractor to determine the reflex measure of $\angle ACU$.



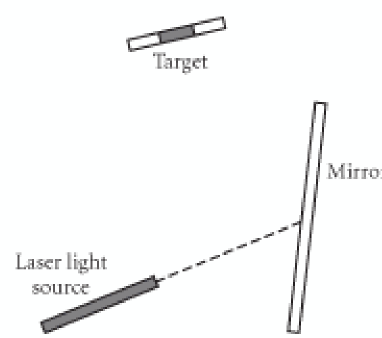
41. Use your protractor to determine the reflex measure of $\angle QUA$.

42. What is the relationship between the measure of an angle and the reflex measure of the angle?

43. If the 4-ball is hit as shown, will it go into the corner pocket? Find the path of the ball using only your protractor and straightedge.



44. The principle you just learned for billiard balls is also true for sound or radio waves bouncing off a surface or for a ray of light reflecting from a mirror. If you hold a laser light angled at the mirror as shown, will the light from the laser hit the target object? Explain.



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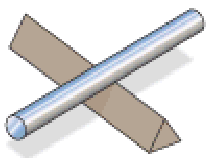
Review

45. If points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$. This is called **segment addition**. Solve the following problem and explain how it represents segment addition.

Podunkville, Smallville, and Gotham City lie along a straight highway with Smallville between the other two towns. If Podunkville and Smallville are 70 km apart and Smallville and Gotham City are 110 km apart, how far apart are Podunkville and Gotham City?

46. Use your ruler to draw a segment with length 12 cm. Then use your ruler to locate the midpoint. Label and mark the figure.

47. The balancing point of an object is called its *center of gravity*. Where is the center of gravity of a thin, rodlike piece of wire or tubing? Copy the thin wire shown below onto your paper. Mark the balance point or center of gravity.



48. Explain the difference between $MS = DG$ and $\overline{MS} \cong \overline{DG}$.

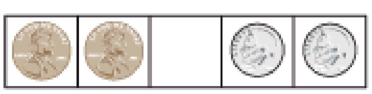
49. Use your ruler and protractor to draw a triangle with angle measures 40° and 70°. Explain your method. Can you draw a second triangle with these two angle measures that looks different from the first?

How is the center of gravity incorporated into the design of this structure?

IMPROVING YOUR VISUAL THINKING SKILLS

Coin Swap I

Arrange two dimes and two pennies on a grid of five squares, as shown. Your task is to switch the position of the two dimes and two pennies in exactly eight moves. A coin can slide into an empty square next to it, or it can jump over one coin into an empty space. Record your solution by drawing eight diagrams that show the moves.



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LESSON 1.3

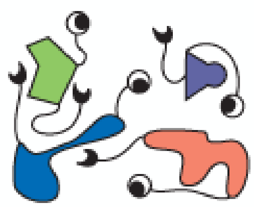
What's a Widget?

Good definitions are very important in geometry. In this lesson you will write your own geometry definitions.

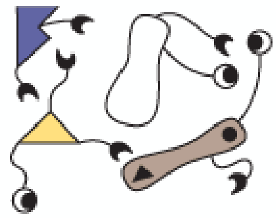
Which creatures in the last group are Widgets?

"When I use a word," Humpty replied in a scornful tone, "it means just what I choose it to mean—neither more nor less." "The question is," said Alice, "whether you can make a word mean so many different things."

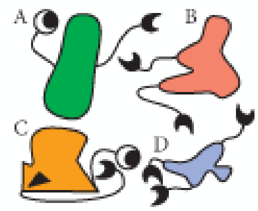
LEWIS CARROLL



Widgets

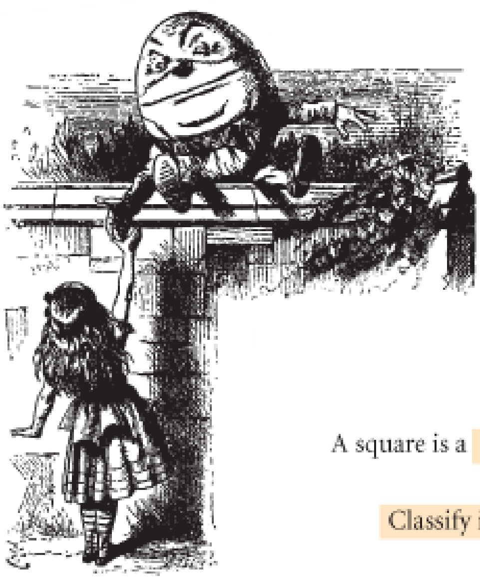


Not Widgets



Which are Widgets?

You might have asked yourself, "What things do all the Widgets have in common, and what things do Widgets have that others do not have?" In other words, what characteristics make a Widget a Widget? They all have colorful bodies with nothing else inside; two tails—one like a crescent moon, the other like an eyeball.



By observing what a Widget is and what a Widget isn't, you identified the characteristics that distinguish a Widget from a non-Widget. Based on these characteristics, you should have selected A as the only Widget in the last group. This same process can help you write good definitions of geometric figures.

This statement defines a protractor: "A protractor is a geometry tool used to measure angles." First, you classify what it is (a geometry tool), then you say how it differs from other geometry tools (it is the one you use to measure angles). What should go in the blanks to define a square?

A square is a ______ that ______.

Classify it. What is it? | How does it differ from others?

Once you've written a definition, you should test it. To do this, you look for a **counter example.** That is, try to create a figure that fits your definition but *isn't* what you're trying to define. If you can come up with a counterexample for your definition, you don't have a good definition.

EXAMPLE A

Everyone knows, "A square is a figure with four equal sides." What's wrong with this definition?

a. Sketch a counterexample. (You can probably find more than one!)

b. Write a better definition for a square.

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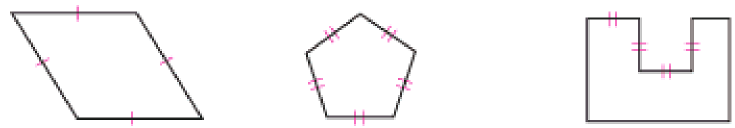
Solution

You probably noticed that "figure" is not specific enough to classify a square, and that "four equal sides" does not specify how it differs from the first counterexample shown below.

a. Three counterexamples are shown here, and you may have found others too.



b. One better definition is "A square is a 4-sided figure that has all sides congruent and all angles measuring 90 degrees."

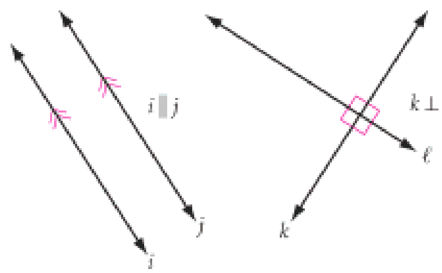
A restaurant counter example

Beginning Steps to Creating a Good Definition

1. **Classify** your term. What is it? ("A square is a 4-sided figure . . .")
2. **Differentiate** your term. How does it differ from others in that class? (". . . that has four congruent sides and four right angles.")
3. **Test** your definition by looking for a counterexample.

Ready to write a couple of definitions? First, here are two more types of markings that are very important in geometry.

The same number of arrow marks indicates that lines are parallel. The symbol $\parallel$ means "is parallel to." A small square in the corner of an angle indicates that it measures 90°. The symbol $\perp$ means "is perpendicular to."



EXAMPLE B

Define these terms:

a. Parallel lines

b. Perpendicular lines

Solution

Following these steps, classify and differentiate each term.

Classify. Differentiate.

a. Parallel lines are lines in the same plane that never meet.

b. Perpendicular lines are lines that meet at 90° angles.

Why do you need to say "in the same plane" for parallel lines, but not for perpendicular lines? Sketch or demonstrate a counterexample to show the following definition is incomplete: "Parallel lines are lines that never meet." (Two lines that do not intersect and are noncoplanar are **skew lines**.)

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Investigation
Defining Angles

Here are some examples and non-examples of special types of angles.

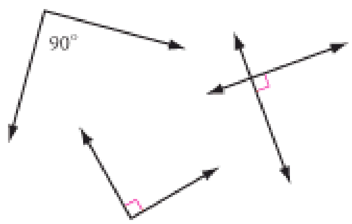
Step 1 Write a definition for each boldfaced term. Make sure your definitions highlight important differences.

Step 2 Trade definitions and test each other's definitions by looking for counterexamples.

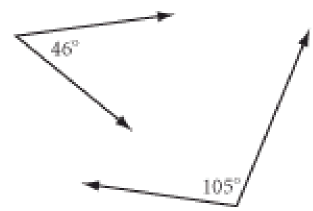
Step 3 If another group member finds a counterexample to one of your definitions, write a better definition. As a group, decide on the best definition for each term.

Step 4 As a class, agree on common definitions. Add these to your notebook. Draw and label a picture to illustrate each definition.

Right Angle



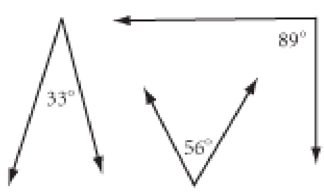
Right angles



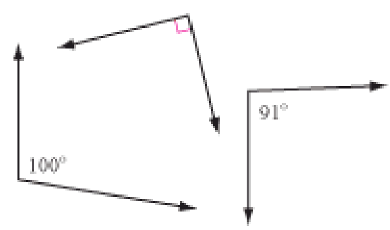
Not right angles

Notice the many congruent angles in this Navajo transitional Wedgeweave blanket. Are they right, acute, or obtuse angles?

Acute Angle

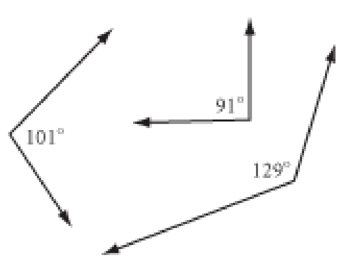


Acute angles

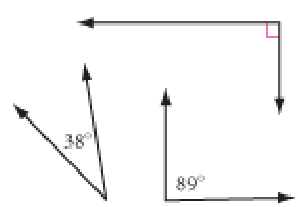


Not acute angles

Obtuse Angle



Obtuse angles



Not obtuse angles

keymath.com/DG

[You can also view the **Dynamic Geometry Exploration** Three Types of Angles at www.keymath.com/DG .]

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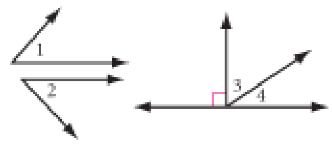
Complementary Angles

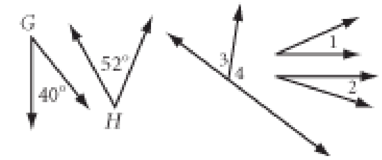
$m\angle 1 + m\angle 2 = 90°$



Pairs of complementary angles:

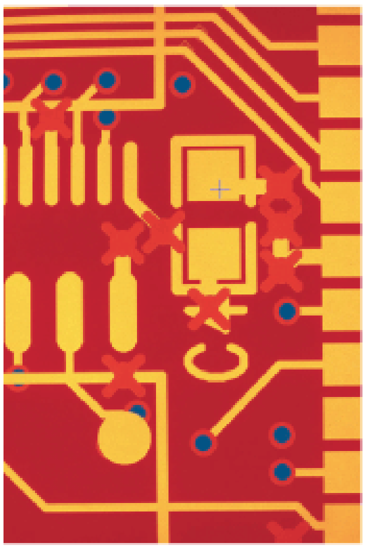
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

$m\angle 1 + m\angle 2 \neq 90°$



Not pairs of complementary angles:

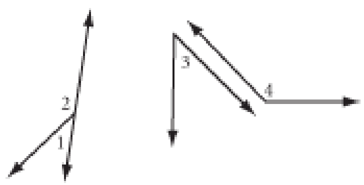
$\angle G$ and $\angle H$ $\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$



What types of angles or angle pairs do you see in this magnified view of a computer chip?

Supplementary Angles

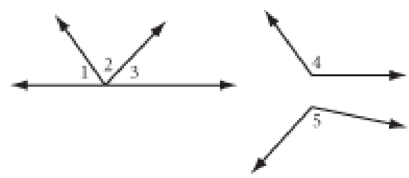
$m\angle 3 + m\angle 4 = 180°$



Pairs of supplementary angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

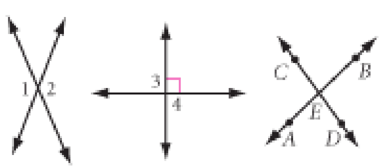
$m\angle 4 + m\angle 5 > 180°$



Not pairs of supplementary angles:

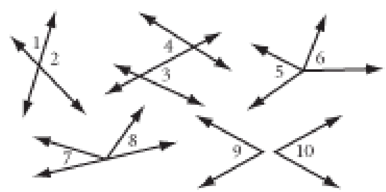
$\angle 1$, $\angle 2$, and $\angle 3$
$\angle 4$ and $\angle 5$

Vertical Angles



Pairs of vertical angles:

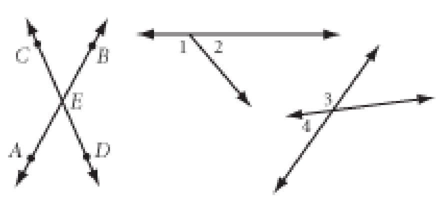
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle AED$ and $\angle BEC$
$\angle AEC$ and $\angle DEB$



Not pairs of vertical angles:

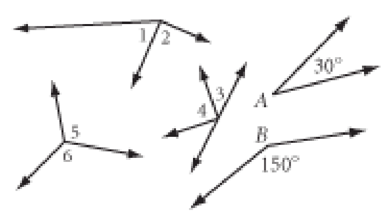
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle 5$ and $\angle 6$
$\angle 7$ and $\angle 8$
$\angle 9$ and $\angle 10$

Linear Pair of Angles



Linear pairs of angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle AED$ and $\angle AEC$
$\angle BED$ and $\angle DEA$



Not linear pairs of angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle 5$ and $\angle 6$
$\angle A$ and $\angle B$

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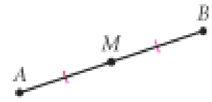
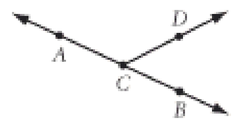
Often geometric definitions are easier to write if you refer to labeled figures. For example, you can define the midpoint of a line segment by saying: "Point *M* is the midpoint of segment *AB* if *M* is a point on segment *AB,* and *AM* is equal to *MB*."



EXAMPLE C | Use a labeled figure to define a linear pair of angles.



▶ Solution | $\angle ACD$ and $\angle BCD$ form a linear pair of angles if point C is on $\overleftrightarrow{AB}$ and lies between points A and B.

Compare this definition with the one you wrote in the investigation. Can there be more than one correct definition?

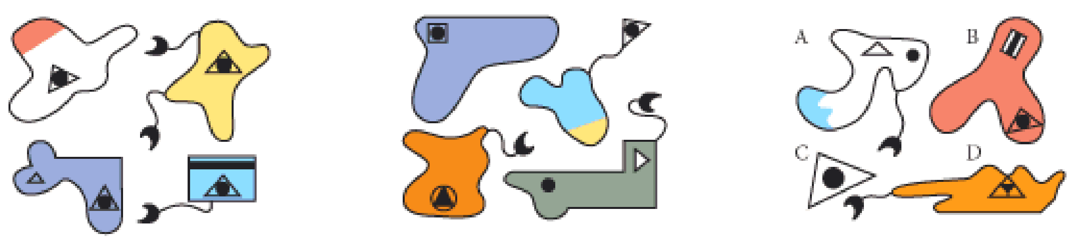
The design of this African Kente cloth contains examples of parallel and perpendicular lines, obtuse and acute angles, and complementary and supplementary angle pairs. To learn about the significance of Kente cloth designs, visit www.keymath.com/DG .

Keymath.com Practice Your Skills

EXERCISES

For Exercises 1–8, draw and carefully label the figures. Use the appropriate marks to indicate right angles, parallel lines, congruent segments, and congruent angles. Use a protractor and a ruler when you need to.

1. Acute angle *DOG* with a measure of 45°
2. Right angle *RTE*
3. Obtuse angle *BIG* with angle bisector $\overleftrightarrow{IE}$
4. $\overleftrightarrow{DG} \parallel \overleftrightarrow{MS}$
5. $\overleftrightarrow{PE} \perp \overleftrightarrow{AR}$
6. Vertical angles *ABC* and *DBE*
7. Complementary angles $\angle A$ and $\angle B$ with $m\angle A = 40°$
8. Supplementary angles $\angle C$ and $\angle D$ with $m\angle D = 40°$
9. Which creatures in the last group below are Zoids? What makes a Zoid a Zoid?



Zoids   Not Zoids   Which are Zoids?

10. What are the characteristics of a good definition?
11. What is the difference between complementary and supplementary angles?

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12. If $\angle X$ and $\angle Y$ are supplementary angles, are they necessarily a linear pair? Why or why not?

13. Write these definitions using the classify and differentiate method to fill in the blanks:

a. An acute angle is __________ that __________.

b. Complementary angles are __________ that __________.

c. A midpoint is __________ that __________.

d. A protractor is __________ that __________.

14. There is something wrong with this definition for a pair of vertical angles: "If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point P, then $\angle APC$ and $\angle BPD$ are a pair of vertical angles." Sketch a counterexample to show why it is not correct. Can you add a phrase to correct it?

For Exercises 15–24, four of the statements are true. Make a sketch or demonstrate each true statement. For each false statement, draw a counterexample.

15. For every line segment there is exactly one midpoint.

16. For every angle there is exactly one angle bisector.

17. If two different lines intersect, then they intersect at one and only one point.

18. If two different circles intersect, then they intersect at one and only one point.

19. Through a given point on a line, there is one and only one line perpendicular to the given line.

20. In every triangle there is exactly one right angle.

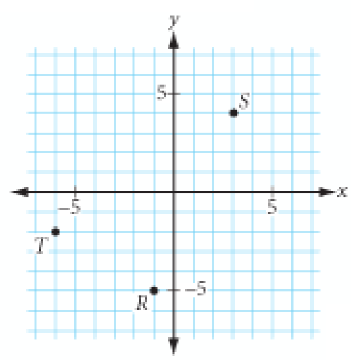
21. Through a point not on a line, one and only one line can be constructed parallel to the given line.

22. If $CA = AT$, then A is the midpoint of $\overline{CT}$.

23. If $m\angle D = 40°$ and $m\angle C = 140°$, then angles C and D are a linear pair.

24. If point A is not the midpoint of $\overline{CT}$, then $CA \neq AT$.

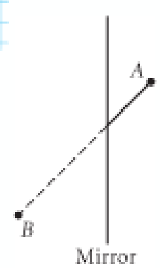
Review



For Exercises 25 and 26, refer to the graph at right.

25. Find possible coordinates of a point P so that points P, T, and S are collinear.

26. Find possible coordinates of a point Q so that $\overleftrightarrow{QR} \parallel \overleftrightarrow{TS}$.



27. A *partial mirror* reflects some light and lets the rest of the light pass through. In the figure at right, half the light from point A passes through the partial mirror to point B. Copy the figure, then draw the outgoing angle for the light reflected from the mirror. What do you notice about the ray of reflected light and the ray of light that passes through?

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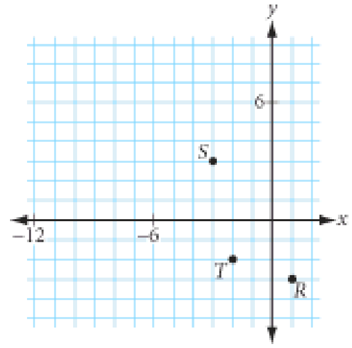
28. Find possible coordinates of points A, B, and C on the graph at right so that $\angle BAC$ is a right angle, $\angle BAT$ is an acute angle, $\angle ABS$ is an obtuse angle, and the points C, T, and R are collinear.



29. If D is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{AB}$, and $AD = 3\text{cm}$, what is the length of $\overline{AB}$?

30. If $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$, $\overrightarrow{BE}$ is the angle bisector of $\angle ABD$, and $m\angle DBC = 24°$, what is $m\angle EBC$?

31. Draw and label a figure that has two congruent segments and three congruent angles. Mark the congruent angles and congruent segments.

32. Show how three lines in a plane can have zero, exactly one, exactly two, or exactly three points of intersection.

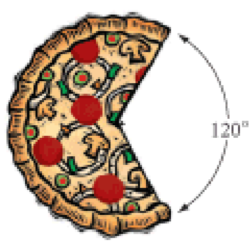
33. Show how it is possible for two triangles to intersect in one point, two points, three points, four points, five points, or six points, but not seven points. Show how they can intersect in infinitely many points.

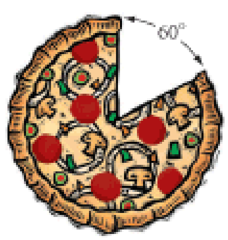
34. Each pizza is cut into slices from the center.

a. What fraction of the pizza is left?

b. What fraction of the pizza is missing?

c. If the pizza is cut into nine equal slices, how many degrees is each angle at the center of the pizza?

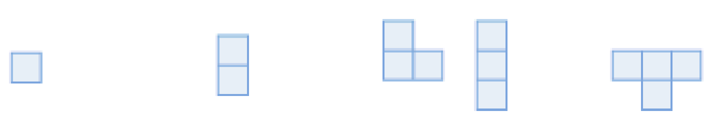




IMPROVING YOUR VISUAL THINKING SKILLS

Polyominoes

In 1953, United States mathematician Solomon Golomb introduced polyominoes at the Harvard Mathematics Club, and they have been played with and enjoyed throughout the world ever since. Polyominoes are shapes made by connecting congruent squares. The squares are joined together side to side. (A complete side must touch a complete side.) Some of the smaller polyominoes are shown below. There is only one monomino and only one domino, but there are two trominoes, as shown. There are five tetrominoes—one is shown. Sketch the other four.



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LESSON

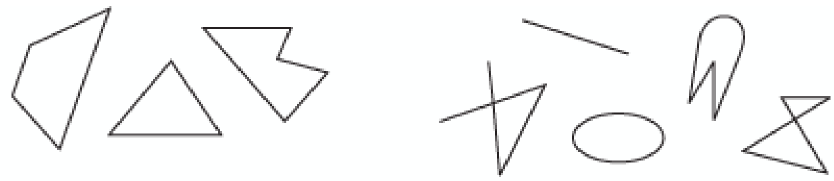
1.4

Polygons

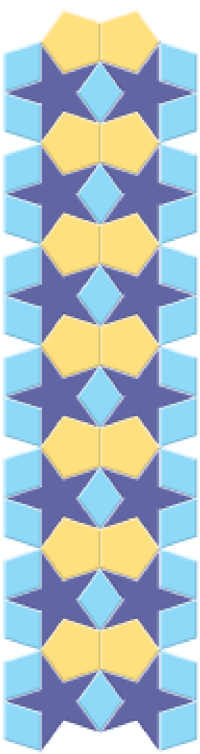
There are two kinds of people in this world: those who divide everything into two groups, and those who don't.

KENNETH BOULDING

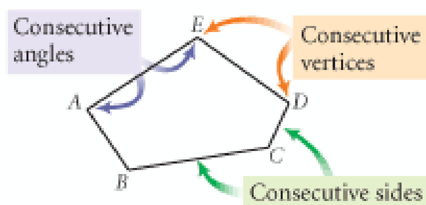
A **polygon** is a closed figure in a plane, formed by connecting line segments endpoint to endpoint with each segment intersecting exactly two others. Each line segment is called a **side** of the polygon. Each endpoint where the sides meet is called a **vertex** of the polygon.



Polygons   Not Polygons

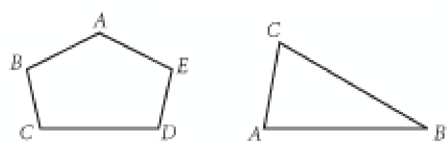


You classify a polygon by the number of sides it has. Familiar polygons have specific names, listed in this table. The ones without specific names are called n-sided polygons, or n-gons. For instance, you call a 25-sided polygon a 25-gon.



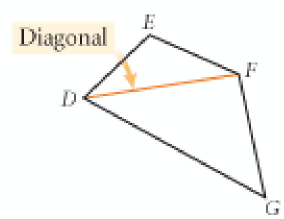
Sides	Name
3	**Triangle**
4	**Quadrilateral**
5	**Pentagon**
6	**Hexagon**
7	**Heptagon**
8	**Octagon**
9	**Nonagon**
10	**Decagon**
11	**Undecagon**
12	**Dodecagon**
n	n-gon

To name a polygon, list the vertices in consecutive order. You can name the pentagon above pentagon *ABCDE*. You can also call it *DCBAE*, but not *BCAED*. When the polygon is a triangle, you use the triangle symbol. For example, $\triangle ABC$ means triangle *ABC*.

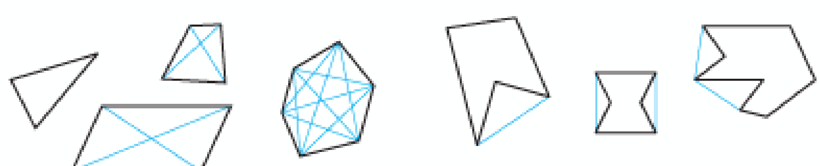


Pentagon *ABCDE* $\triangle ABC$

A **diagonal** of a polygon is a line segment that connects two nonconsecutive vertices.



A polygon is **convex** if no diagonal is outside the polygon. A polygon is **concave** if at least one diagonal is outside the polygon.



Convex polygons: All diagonals are inside   Concave polygons: One or more diagonals are outside

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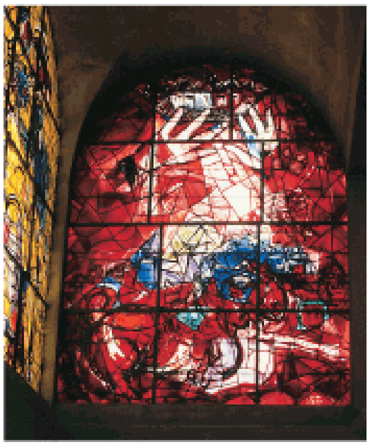
Recall that two segments or two angles are congruent if and only if they have the same measures. Two polygons are **congruent** if and only if they are exactly the same size and shape. "If and only if" means that the statements work both ways.

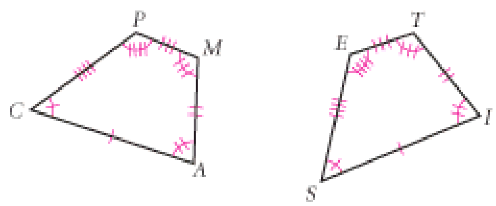


How does the shape of the framework of this Marc Chagall (1887–1985) stained glass window support the various shapes of the design?

If polygons are congruent, then corresponding sides and angles are congruent.

If corresponding sides and angles are congruent, then polygons are congruent.

For example, if quadrilateral *CAMP* is congruent to quadrilateral *SITE,* then their four pairs of corresponding angles and four pairs of corresponding sides are also congruent. When you write a statement of congruence, always write the letters of the corresponding vertices in an order that shows the correspondences.

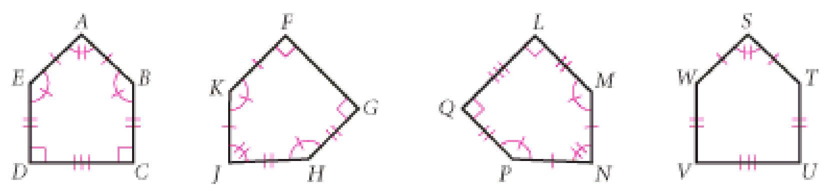


$CAMP \cong SITE$

EXAMPLE

Which polygon is congruent to *ABCDE*?

$ABCDE \cong \underline{\ ?\ }$



▶ **Solution**

Polygons *JKFGH* and *ABCDE* have all corresponding angles congruent, but not all corresponding sides. Polygons *STUVW* and *ABCDE* have all corresponding sides congruent, but not all corresponding angles.

All corresponding sides and angles must be congruent, so $ABCDE \cong NPQLM$.

You could also say $ABCDE \cong NMLQP$ because all the congruent parts would still match.

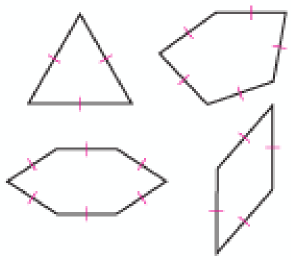
The **perimeter** of a polygon equals the sum of the lengths of its sides. Perimeter measures the length of the boundary of a two-dimensional figure.

The quadrilateral at right has perimeter 37 cm.

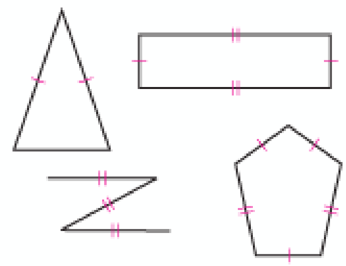
Investigation
Special Polygons

Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions for your class and add them to your definitions list. In your notebook, draw and label a figure to illustrate each definition.

Equilateral Polygon

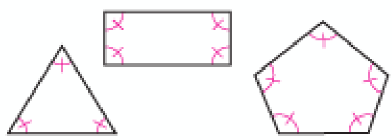


Equilateral polygons

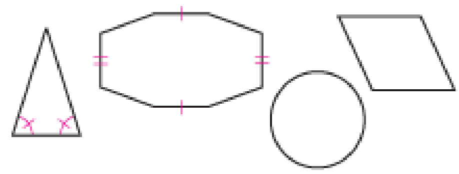


Not equilateral polygons

Equiangular Polygon

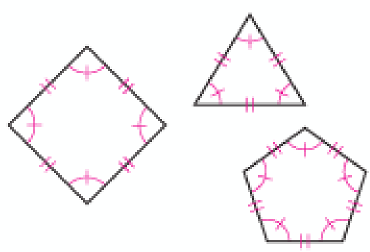


Equiangular polygons

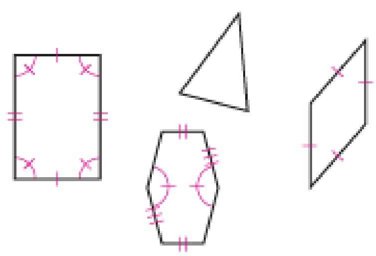


Not equiangular polygons

Regular Polygon



Regular polygons



Not regular polygons

EXERCISES

For Exercises 1–3, draw an example of each polygon.

1. Quadrilateral **2.** Dodecagon **3.** Octagon

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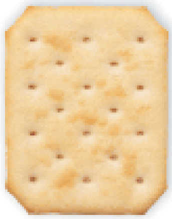
For Exercises 4–7, classify each polygon. Assume that all sides are straight.

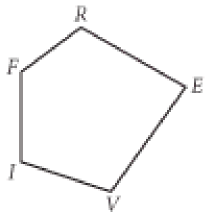
4. 

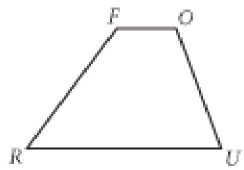
5. 

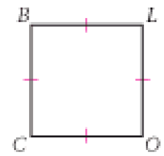
6. 

7. 

For Exercises 8–10, give one possible name for each polygon.

8. 

9. 

10. 

11. Write these definitions using the classify and differentiate method to fill in the blanks:

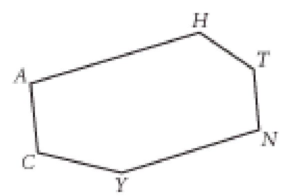
a. An octagon is __________ that ___________.

b. A concave polygon is __________ that ___________.

c. A 20-gon, also called an icosagon, is __________ that ___________.

d. An equilateral polygon is __________ that ___________.

12. Name a pair of consecutive angles and a pair of consecutive sides in the figure at right.

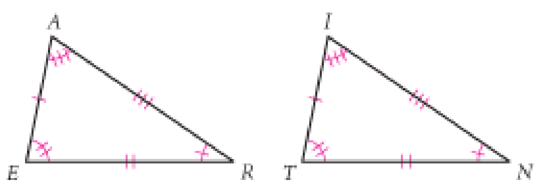


13. Draw a concave hexagon. How many diagonals does it have?

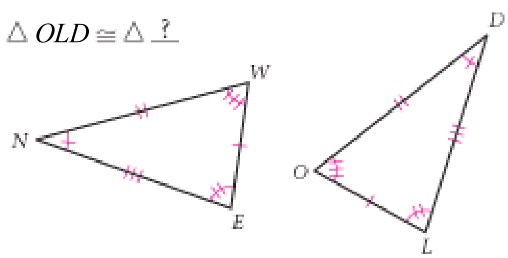
14. Name the diagonals of pentagon *ABCDE.*

For Exercises 15 and 16, use the information given to name the triangle that is congruent to the first one.

15. $\triangle EAR \cong \triangle$ _?_ ⓗ



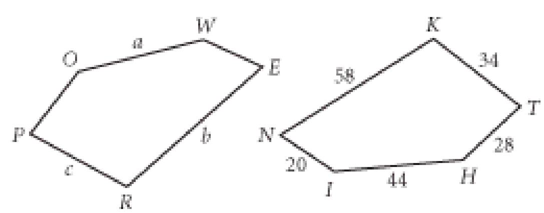
16. $\triangle OLD \cong \triangle$ _?_



17. In the figure at right, *THINK* $\cong$ *POWER.*

a. Find the measures *a, b,* and *c*.

b. If $m\angle P = 87°$ and $m\angle W = 165°$, which angles in *THINK* do you know? Write their measures.



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18. If pentagon *FIVER* is congruent to pentagon *PANCH,* then which side in pentagon *FIVER* is congruent to side $\overline{PA}$? Which angle in pentagon *PANCH* is congruent to $\angle IVE$?

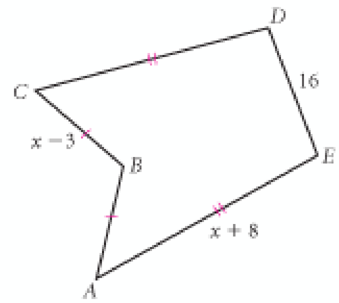
19. Use your geometry tools to draw a convex hexagon with two consecutive sides measuring 5 cm and three consecutive angles measuring 130°.

20. Draw an equilateral concave pentagon. Then draw an equiangular convex pentagon.

21. Each side of a regular dodecagon measures 7 in. Find the perimeter.

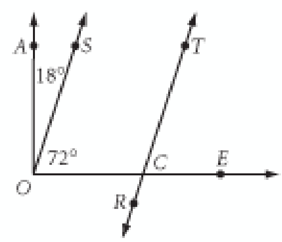
22. The perimeter of an equilateral octagon is 42 cm. Find the length of each side.

23. The perimeter of *ABCDE* is 94 m. Find the lengths of segments *AB* and *CD*.



Review

24. Name a pair of complementary angles and a pair of vertical angles in the figure at right.



25. Draw $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ with $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$.

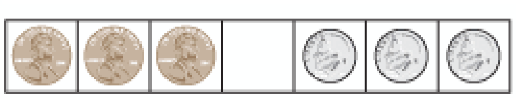
26. Draw a counterexample to show that this statement is false: "If a rectangle has perimeter 50 meters, then a pair of consecutive sides measures 10 meters and 15 meters."

27. Is it possible for four lines in a plane to have exactly zero points of intersection? One point? Two points? Three points? Four points? Five points? Six points? Draw a figure to support each of your answers.

IMPROVING YOUR VISUAL THINKING SKILLS

Coin Swap II

Arrange three dimes and three pennies on a grid of seven squares, as shown. Follow the same rules as in Coin Swap I on page 46 to switch the position of the three dimes and three pennies in exactly 15 moves. Record your solution by listing in order which coin is moved. For example, your list might begin PDP. . . .



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LESSON

1.5

Triangles

The difference between the right word and the almost right word is the difference between lightning and the lightning bug.

MARK TWAIN

You have learned to be careful with geometry definitions. It turns out that you also have to be careful with diagrams.

When you look at a diagram, be careful not to assume too much from it. To **assume** something is to accept it as true without facts or proof.

Lightning

Not lightning

Things you may assume:

You may assume that lines are straight, and if two lines intersect, they intersect at one point.

You may assume that points on a line are collinear and that all points shown in a diagram are coplanar unless planes are drawn to show that they are noncoplanar.

Things you may not assume:

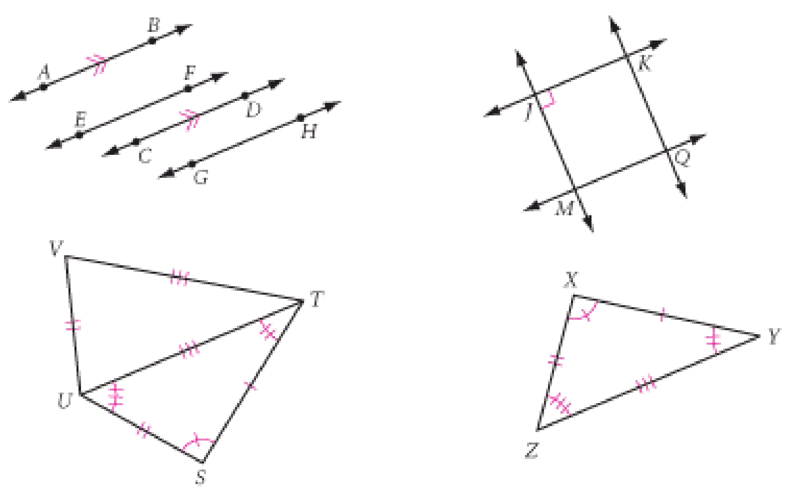
You may not assume that just because two lines or segments *look* parallel that they *are* parallel—they must be *marked* parallel!

You may not assume that two lines *are* perpendicular just because they *look* perpendicular—they must be *marked* perpendicular!

Pairs of angles, segments, or polygons are not necessarily congruent unless they are *marked* with information that tells you they must be congruent!

EXAMPLE

In the diagrams below, which pairs of lines are perpendicular? Which pairs of lines are parallel? Which pair of triangles is congruent?



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Solution By studying the markings, you can tell that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{JK} \perp \overleftrightarrow{JM}$, and $\triangle STU \cong \triangle XYZ$.

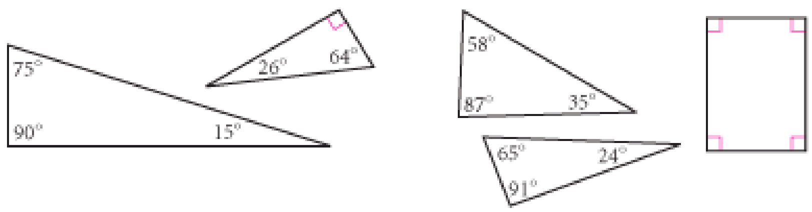
In this lesson you will write definitions that classify different kinds of triangles based on relationships among their sides and angles.

Investigation
Triangles

Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions for your class and add them to your definition list. In your notebook, draw and label a figure to illustrate each definition.

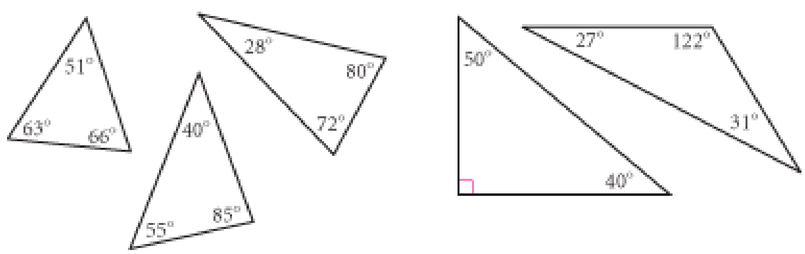
What shape is the basis for the design on this textile from Uzbekistan?

Right Triangle



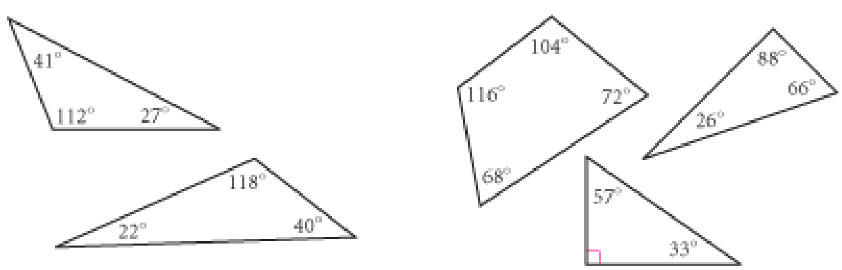
Right triangles Not right triangles

Acute Triangle



Acute triangles Not acute triangles

Obtuse Triangle



Obtuse triangles Not obtuse triangles

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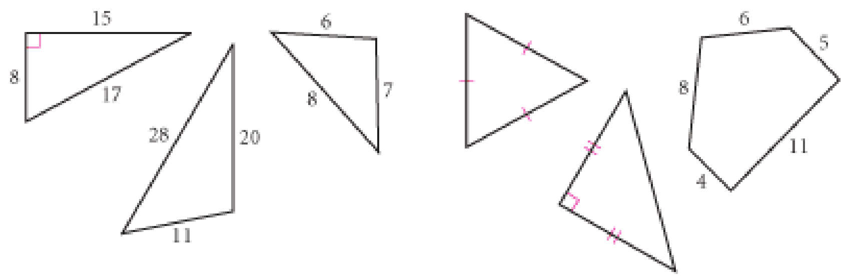
The Sol LeWitt (b 1928, United States) design inside this art museum uses triangles and quadrilaterals to create a painting the size of an entire room.

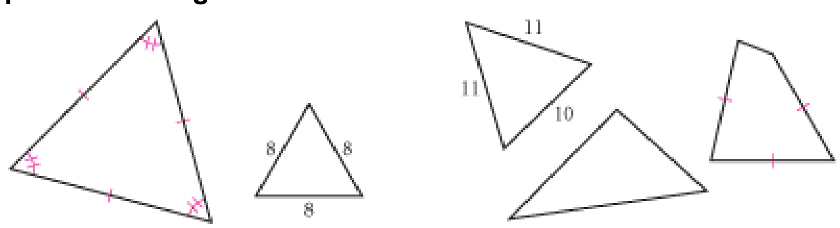
Sol LeWitt, Wall Drawing #652—On three walls, continuous forms with color ink washes superimposed, color in wash. Collection: Indianapolis Museum of Art, Indianapolis, IN. September, 1990. Courtesy of the artist.

Scalene Triangle



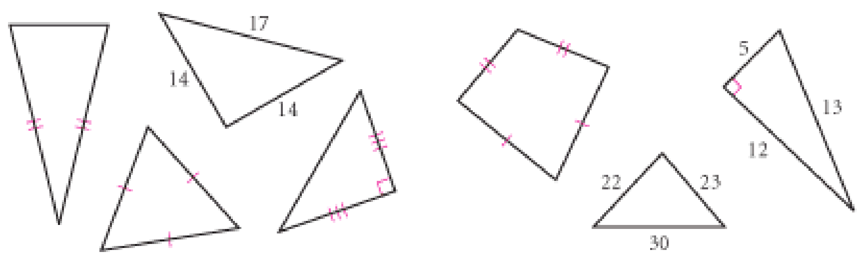
Scalene triangles   Not scalene triangles

Equilateral Triangle



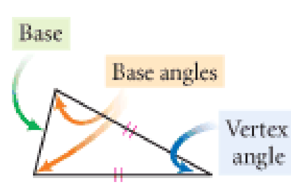
Equilateral triangles   Not equilateral triangles

Isosceles Triangle



Isosceles triangles   Not isosceles triangles

In an isosceles triangle, the angle between the two sides of equal length is called the **vertex angle.** The side opposite the vertex angle is called the **base** of the isoceles triangle. The two angles opposite the two sides of equal length are called the **base angles** of the isoceles triangle.



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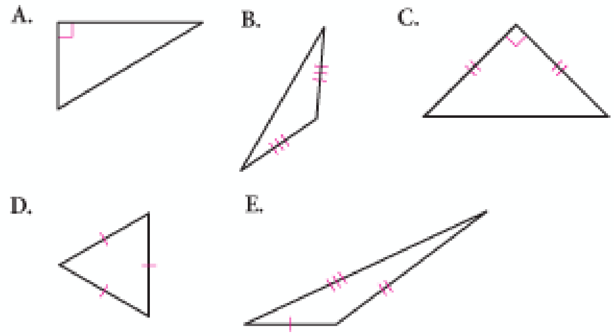
EXERCISES

For Exercises 1–4, match the term on the left with its figure on the right.

1. Equilateral triangle
2. Scalene right triangle
3. Isosceles right triangle
4. Isosceles obtuse triangle



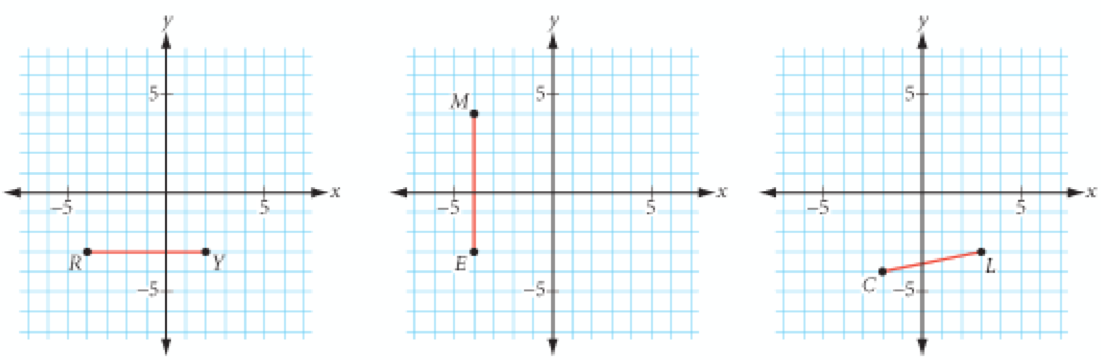
For Exercises 5–9, sketch and label the figure. Mark the figures.

5. Isosceles acute triangle ACT with $AC = CT$
6. Scalene triangle SCL with angle bisector $\overline{CM}$
7. Isosceles right triangle CAR with $m\angle CRA = 90°$
8. Two different isosceles triangles with perimeter $4a + b$
9. Two noncongruent triangles, each with side 6 cm and an angle measuring 40°

10. Use your ruler and protractor to draw an isosceles acute triangle with base AC and vertex angle B.

11. Use your ruler and protractor to draw an isosceles obtuse triangle ZAP with base angles A and Z.

For Exercises 12–14, use the graphs below. Can you find more than one answer?



12. Locate a point L so that $\triangle LRY$ is an isosceles triangle.
13. Locate a point O so that $\triangle MOE$ is an isosceles right triangle.
14. Locate a point R so that $\triangle CRL$ is an isosceles right triangle.

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15. Use your ruler and protractor to draw a triangle with one side 9 cm long and an adjacent angle measuring 45°. Explain your method. Can you draw a second triangle with the given measures that is not congruent to the first?

16. Use your ruler and protractor to draw a triangle with one angle measuring 40° and an opposite side 10 cm long. Explain your method. Can you draw a second triangle with the given measures that is not congruent to the first?

Review

For Exercises 17–21, tell whether the statement is true or false. For each false statement, sketch a counterexample or explain why the statement is false.

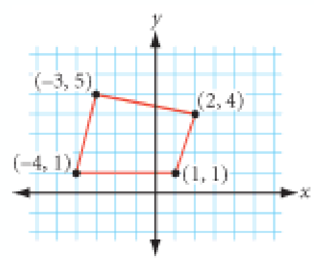
17. An acute angle is an angle whose measure is less than 90°.

18. If two lines intersect to form a right angle, then the lines are perpendicular.

19. A diagonal is a line segment that connects any two vertices of a polygon.

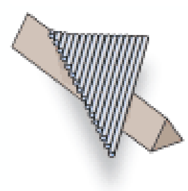
20. A ray that divides the angle into two angles is the angle bisector.

21. An obtuse triangle has exactly one angle whose measure is greater than 90°.



22. Use the ordered pair rule $(x, y) \rightarrow (x + 1, y - 3)$ to relocate the four vertices of the given quadrilateral. Connect the four new points to create a new quadrilateral. Do the two quadrilaterals appear congruent? Check your guess with tracing paper or patty paper.

23. Suppose a set of thin rods is glued together into a triangle as shown. How would you place the triangular arrangement of rods onto the edge of a ruler so that they balance? Explain why.



For Exercises 24–26, sketch and carefully label the figure. Mark the congruent parts.

24. Pentagon *PENTA* with $PE = EN$

25. Hexagon *NGAXEH* with $\angle HEX \cong \angle EXA$

26. Equiangular quadrilateral *QUAD* with $QU \neq QD$

IMPROVING YOUR VISUAL THINKING SKILLS

Pentominoes I

In Polyominoes, you learned about shapes called polyominoes. Polyominoes with five squares are called pentominoes. Can you find all possible pentominoes? One is shown at right. Use graph paper or square dot paper to sketch them.

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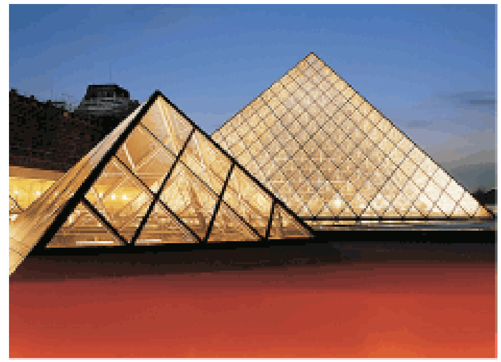
LESSON 1.6

Special Quadrilaterals

If you don't live it, it won't come out of your horn.
CHARLIE PARKER

If you attach two congruent triangles, you create many different quadrilaterals that have special properties. For example, the quadrilaterals in the photo at right can be formed by reflecting an isosceles triangle across its base, resulting in a quadrilateral with four equal sides. In this lesson you will define different types of special quadrilaterals based on relationships of their sides and angles.



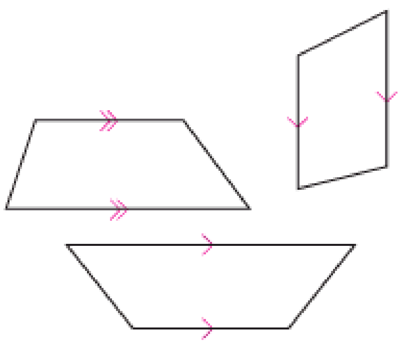
How many shapes make up the overall triangular shapes of these pyramids at the Louvre in Paris?

Investigation

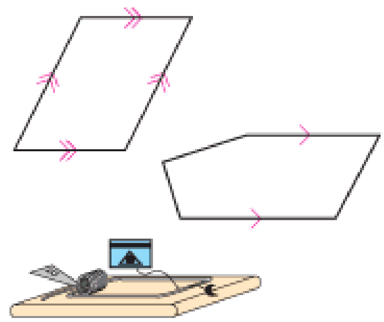
Special Quadrilaterals

Write a good definition of each boldfaced term.Discuss your definitions with others in your group.Agree on a common set of definitions for your class and add them to your definitions list. In your notebook, draw and label a figure to illustrate each definition.

Trapezoid

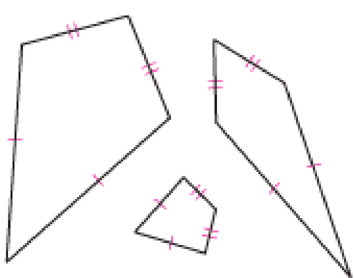


Trapezoids

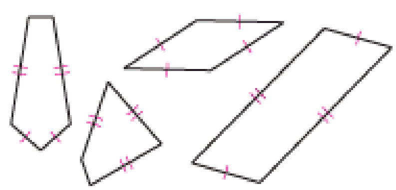


Not trapezoids

Kite



Kites



Not kites

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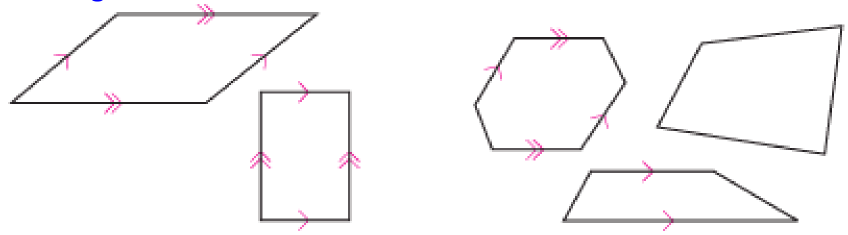
Recreation CONNECTION

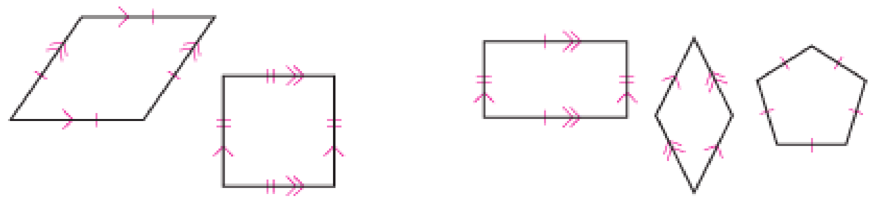
Today's kite designers use lightweight synthetic fabrics and complex shapes to sustain kites in the air longer than earlier kites made of wood and cloth that had the basic "kite" shape. Many countries hold annual kite festivals.

Parallelogram



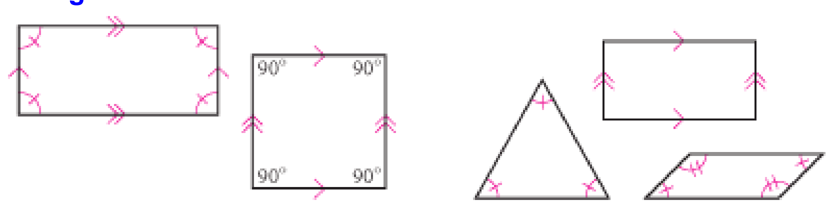
Parallelograms Not parallelograms

Rhombus



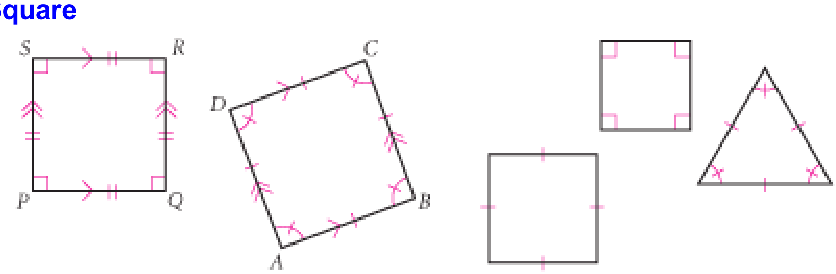
Rhombuses Not rhombuses

Rectangle



Rectangles Not rectangles

Square



Squares Not squares

As you learned in the investigation, a figure that looks like a square is not a square unless it has the proper markings. Keep this in mind as you work on the exercises.

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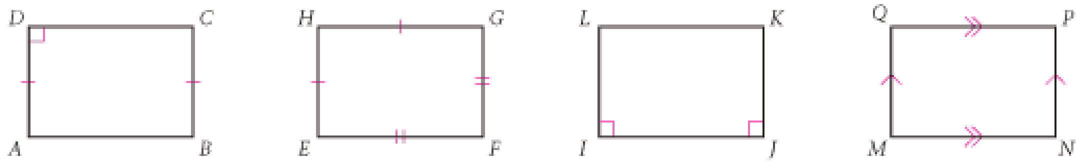
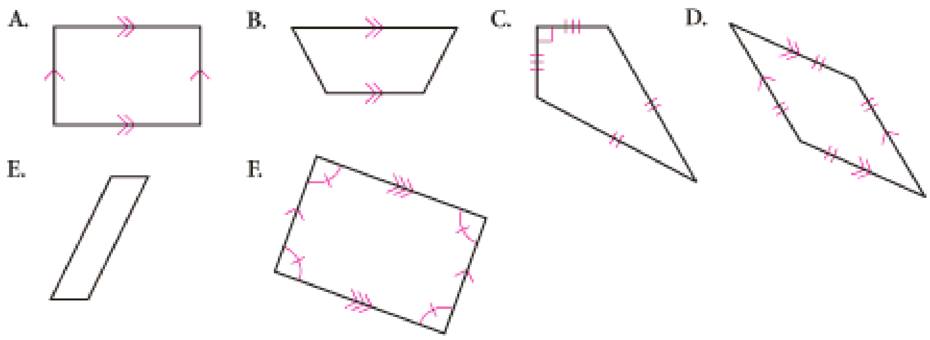
EXERCISES

1. Based on the marks, what can you assume to be true in each figure?



For Exercises 2–6, match the term on the left with its figure on the right.

2. Trapezoid
3. Rhombus
4. Rectangle
5. Kite
6. Parallelogram



For Exercises 7–10, sketch and label the figure. Mark the figures.

7. Trapezoid *ZOID* with $\overline{ZO} \parallel \overline{ID}$

8. Kite *BENF* with $BE = EN$

9. Rhombus *EQUL* with diagonals $\overline{EU}$ and $\overline{QL}$ intersecting at *A*

10. Rectangle *RGHT* with diagonals $\overline{RH}$ and $\overline{GT}$ intersecting at *I*

11. Draw a hexagon with exactly two outside diagonals.

12. Draw a regular quadrilateral. What is another name for this shape?

13. Find the other two vertices of a square with one vertex (0, 0) and another vertex (4, 2). Can you find another answer?

Architecture CONNECTION

Quadrilaterals are used in the architecture of many cultures for both practical purposes and aesthetic appeal. The Acoma Pueblo Dwellings in New Mexico, the Chichén Itzá pyramid in Mexico, and the spiral staircase in an apartment house designed by Austrian architect and artist Friedensreich Hundertwasser (1928–2000) all use quadrilateral-based designs for constructing climbing structures and enhancing overall attractiveness.

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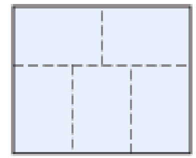
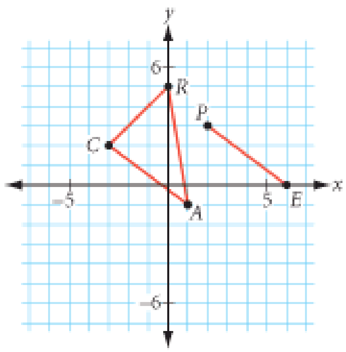
14. A rectangle with perimeter 198 cm is divided into five congruent rectangles, as shown in the diagram at right. What is the perimeter of one of the five congruent rectangles? (h)

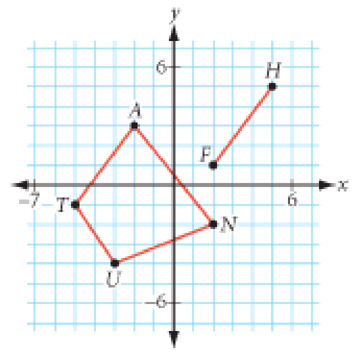


For Exercises 15–18, copy the given polygon and segment onto graph paper. Give the coordinates of the missing points.

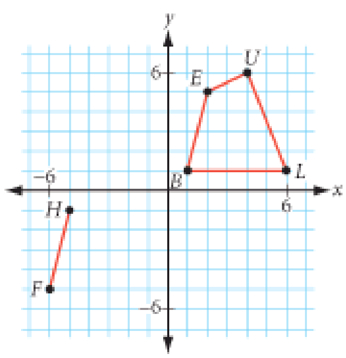
15. $\triangle CAR \cong \triangle PET$



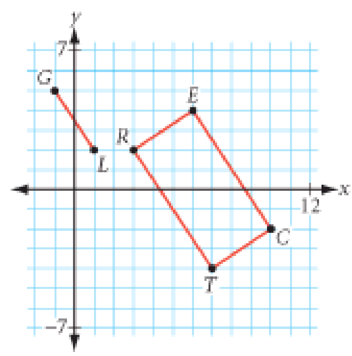
16. $TUNA \cong FISH$



17. $BLUE \cong FISH$



18. $RECT \cong ANGL$



19. Draw and cut out two congruent acute scalene triangles.

a. Arrange them into a kite. Sketch the result and mark all congruent sides.

b. Arrange them into a parallelogram. Sketch the result and mark all congruent sides.

20. Draw and cut out two congruent obtuse isosceles triangles. Which special quadrilaterals can you create with these two congruent triangles? Explain.

21. Imagine using two congruent triangles to create a special quadrilateral, as you did in the last two exercises.

a. What type of triangles do you need to form a rectangle? Explain.

b. What type of triangles do you need to form a square? Explain.

The repeating pattern of squares and triangles creates a geometric tree in this quilt design by Diane Venters. What other polygons can you find in this quilt?

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Review

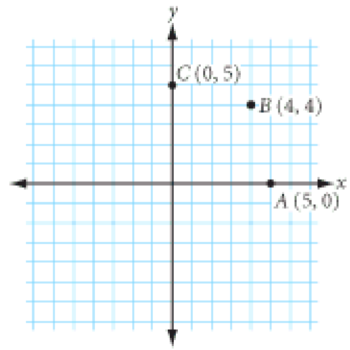
For Exercises 22–24, sketch and carefully label the figure. Mark the congruent parts.

22. A hexagon with exactly one line of reflectional symmetry

23. Two different equilateral pentagons with perimeter 25 cm

24. Use your compass, protractor, and straightedge to draw a regular pentagon.

25. Draw an equilateral octagon *ABCDEFGH* with $A(5, 0)$, $B(4, 4)$, and $C(0, 5)$ as three of its vertices. Is it regular?



project

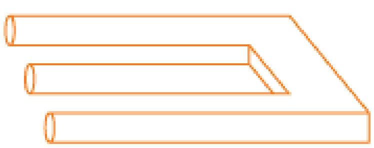
DRAWING THE IMPOSSIBLE

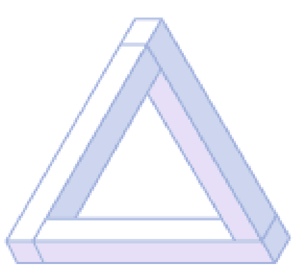
Some optical illusions are tricks—they at first appear to be drawings of real objects, but actually they are impossible to make, except on paper.

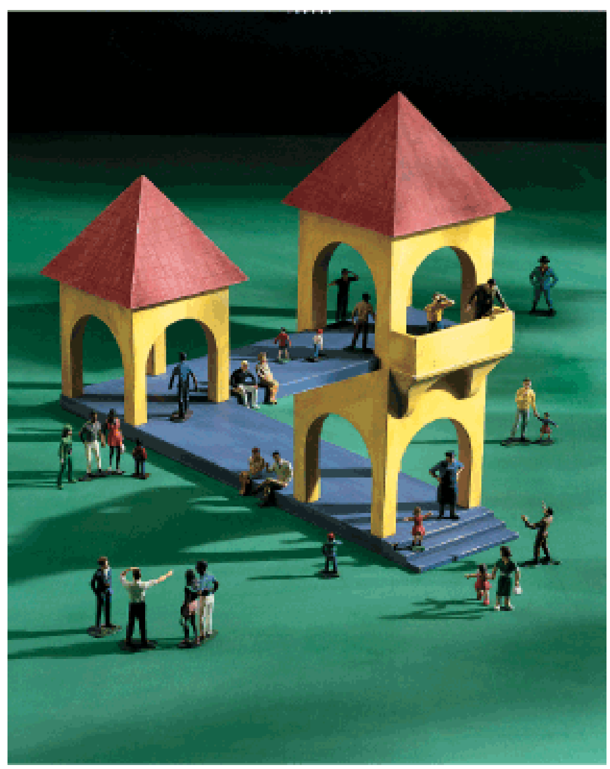
For instance, see the photograph and drawings shown here, and the two pieces by M. C. Escher on p. 421 and p. 477. Try drawing some impossible objects. First, copy these two impossible objects by drawing them on full sheets of paper. Then create one of your own, either in a drawing or photograph.

Your project should include

- The two impossible drawings below.
- Your own impossible drawing or photograph.







From WALTER WICK'S OPTICAL TRICKS. Published by Cartwheel Books, a division of Scholastic Inc. ©1998 by Walter Wick. Reprinted by permission.

To see more examples or to further explore optical illusions, visit www.keymath.com/DG .

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LESSON 1.7

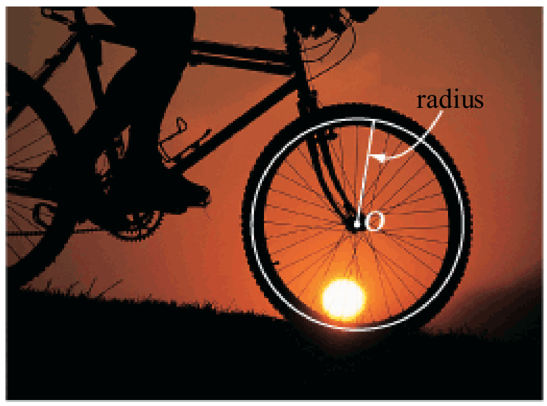
Circles

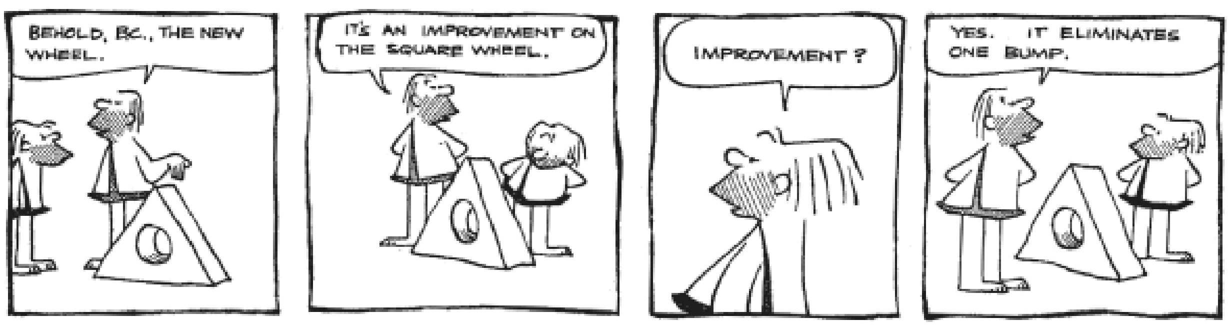
I can never remember things I didn't understand in the first place.
AMY TAN

Unless you walked to school this morning, you arrived on a vehicle with circular wheels.

A **circle** is the set of all points in a plane at a given distance from a given point. The given distance is called the **radius** and the given point is called the **center.** You name a circle by its center. The circle on the bicycle wheel, with center *O,* is called circle *O.* When you see a dot at the center of a circle, you can assume that it represents the center point.

A segment from the center to a point on the edge of the circle is called a radius. Its length is also called the radius. A bicycle wheel is a physical model of a circle, and one spoke is a close physical model of a radius.





By permission of Johnny Hart and Creators Syndicate, Inc.

Science CONNECTION

A pebble dropped in a pond sends out circular ripples. These waves radiate from the point where the pebble hits the water in all directions at the same speed, so every point is equally distant from the center. This unique property of circles appears in many other real-world contexts, such as radio waves sent from an antenna, seismic waves moving from the center of an earthquake, or sand draining out of a hole.

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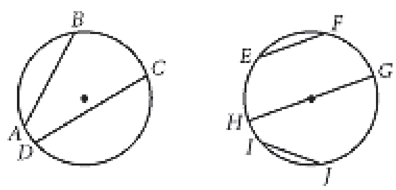
Investigation

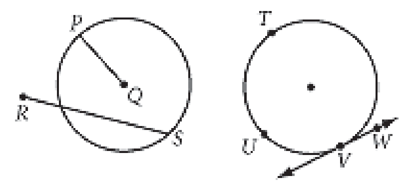
Defining Circle Terms

Step 1 Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions as a class and add them to your definition list. In your notebook, draw and label a figure to illustrate each definition.

Chord

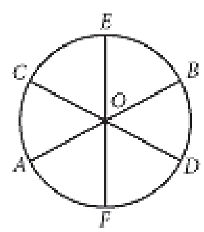


Chords:
$\overline{AB}$, $\overline{CD}$, $\overline{EF}$, $\overline{GH}$, and $\overline{IJ}$

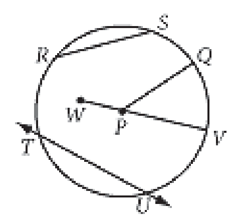


Not chords:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overline{VW}$

Diameter

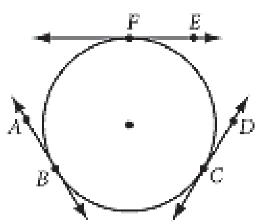


Diameters:
$\overline{AB}$, $\overline{CD}$, and $\overline{EF}$

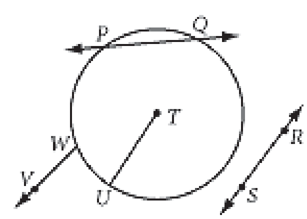


Not diameters:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overline{VW}$

Tangent



Tangents:
$\overline{AB}$, $\overline{CD}$, and $\overline{EF}$



Not tangents:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overline{VW}$

Note: You can say $\overleftrightarrow{AB}$ is a tangent, or you can say $\overleftrightarrow{AB}$ is tangent to circle O. The point where the tangent touches the circle is called the **point of tangency.**

Step 2 Can a chord of a circle also be a diameter of the circle? Can it be a tangent? Explain why or why not.

Step 3 Can two circles be tangent to the same line at the same point? Draw a sketch and explain.

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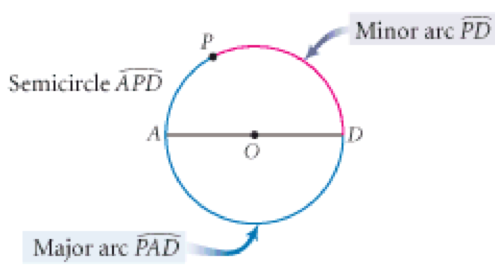
If two or more circles have the same radius, they are **congruent circles.** If two or more coplanar circles share the same center, they are **concentric circles.** All the CDs represent congruent circles, but if you look closely at each CD, you can also see concentric circles.

Congruent circles

Concentric circles

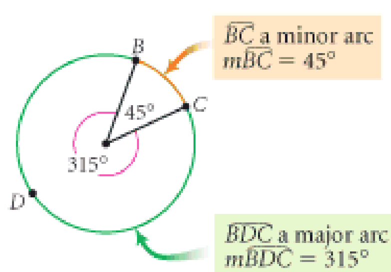
An **arc** of a circle is two points on the circle and a continuous (unbroken) part of the circle between the two points. The two points are called the **endpoints** of the arc.

You write arc AB as $\overset{\frown}{AB}$ or $\overset{\frown}{BA}$. You classify arcs into three types: semicircles, minor arcs, and major arcs. A **semicircle** is an arc of a circle whose endpoints are the endpoints of a diameter. A **minor arc** is an arc of a circle that is smaller than a semicircle. A **major arc** is an arc of a circle that is larger than a semicircle. You can name minor arcs with the letters of the two endpoints. For semicircles and major arcs, you need three points to make clear which arc you mean—the first and last letters are the endpoints and the middle letter is any other point on the arc.



Try to name another minor arc and another major arc in this diagram. Why are three letters needed to name a major arc?

Arcs have a degree measure, just as angles do. A full circle has an arc measure of 360°, a semicircle has an arc measure of 180°, and so on. The **arc measure** of a minor arc is the same as the measure of the **central angle**, the angle with its vertex at the center of the circle, and sides passing through the endpoints of the arc. The measure of a major arc is the same as the reflex measure of the central angle.



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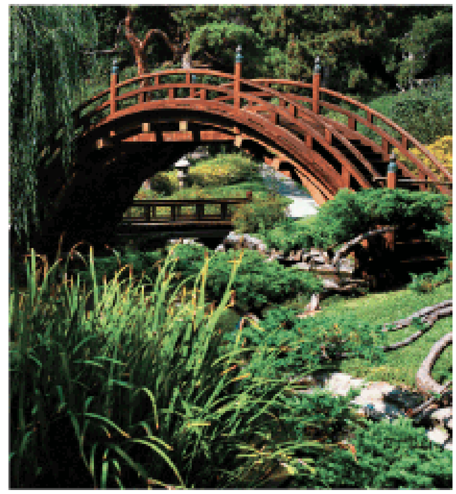
EXERCISES

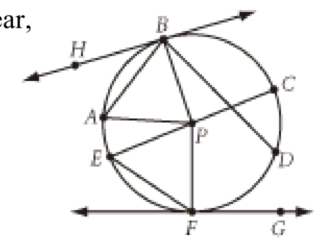
1. In the photos below, identify the physical models that represent a circle, a radius, a chord, a tangent, and an arc of a circle.

Circular irrigation on a farm

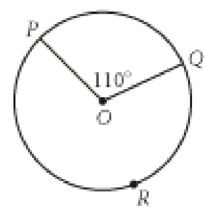


Japanese wood bridge

For Exercises 2–9, use the diagram at right. Points *E, P,* and *C* are collinear, and *P* is the center of the circle.



2. Name three chords.
3. Name one diameter.
4. Name five radii.
5. Name five minor arcs.
6. Name two semicircles.
7. Name two major arcs.
8. Name two tangents.
9. Name a point of tangency.



10. Name two types of vehicles that use wheels, two household appliances that use wheels, and two uses of the wheel in the world of entertainment.

11. In the figure at right, what is $m\overset{\frown}{PQ}$? $m\overset{\frown}{PRQ}$?

12. Use your compass and protractor to make an arc with measure 65°. Now make an arc with measure 215°. Label each arc with its measure.

13. Name two places or objects where concentric circles appear. Bring an example of a set of concentric circles to class tomorrow. You might look in a magazine for a photo or make a copy of a photo from a book (but not this book!).

14. Sketch two circles that appear to be concentric. Then use your compass to construct a pair of concentric circles.

15. Sketch circle *P*. Sketch a triangle inside circle *P* so that the three sides of the triangle are chords of the circle. This triangle is "inscribed" in the circle. Sketch another circle and label it *Q*. Sketch a triangle in the exterior of circle *Q* so that the three sides of the triangle are tangents of the circle. This triangle is "circumscribed" about the circle.

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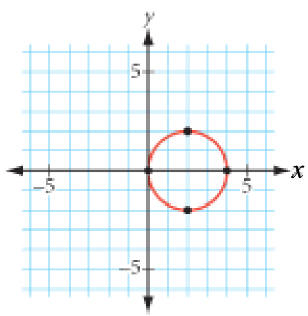
16. Use your compass to construct two circles with the same radius intersecting at two points. Label the centers P and Q. Label the points of intersection of the two circles A and B. Construct quadrilateral $PAQB$. What type of quadrilateral is it?

17. Do you remember the daisy construction from Chapter 0? Construct a circle with radius s. With the same compass setting, divide the circle into six congruent arcs. Construct the chords to form a regular hexagon inscribed in the circle. Construct radii to each of the six vertices. What type of triangle is formed? What is the ratio of the perimeter of the hexagon to the diameter of the circle?

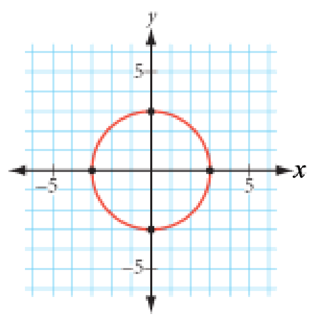
18. Sketch the path made by the midpoint of a radius of a circle if the radius is rotated about the center.

For Exercises 19–21, use the ordered pair rule shown to relocate the four points on the given circle. Can the four new points be connected to create a new circle? Does the new figure appear congruent to the original circle?

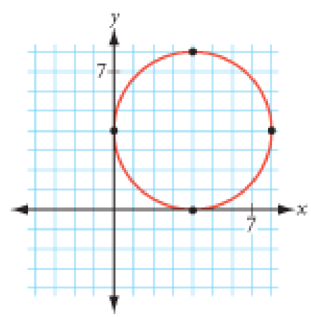
19. $(x, y) \rightarrow (x - 1,\ y + 2)$



20. $(x, y) \rightarrow (2x, 2y)$ ⓗ



21. $(x, y) \rightarrow (2x, y)$



Review

22. If point D is in the interior of $\angle CAB$, thenk $m\angle CAD + m\angle DAB = m\angle CAB$. called **angle addition.** Solve the following problem and explain how it is related to angle addition.

You have a slice of pizza with a central angle that measures 140° that you want to share with your friend. She cuts it through the vertex into two slices. You choose one slice that measures 60°. How many degrees are in the other slice?

For Exercises 23–26, draw each kind of triangle or write "not possible" and explain why. Use your geometry tools to make your drawings as accurate as possible.

23. Isosceles right triangle

24. Scalene isosceles triangle

25. Scalene obtuse triangle

26. Isosceles obtuse triangle

27. Earth takes 365.25 days to travel one full revolution around the Sun. By approximately how many degrees does the Earth travel each day in its orbit around the Sun?

28. Earth completes one full rotation each day, making the Sun appear to rise and set. If the Sun passes directly overhead, by how many degrees does its position in the sky change every hour?

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For Exercises 29–37, sketch, label, and mark the figure or write "not possible" and explain why.

29. Obtuse scalene triangle *FAT* with $m\angle FAT = 100°$

30. Trapezoid *TRAP* with $\overline{TR} \parallel \overline{AP}$ and $\angle TRA$ a right angle

31. Two different (noncongruent) quadrilaterals with angles of 60°, 60°, 120°, and 120°

32. Equilateral right triangle

33. Right isosceles triangle *RGT* with $RT = GT$ and $m\angle RTG = 90°$

34. An equilateral triangle with perimeter $12a + 6b$

35. Two triangles that are not congruent, each with angles measuring 50° and 70°

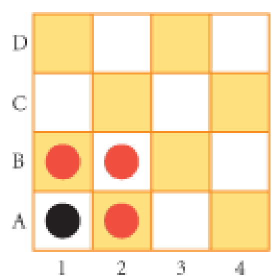
36. Rhombus *EQUI* with perimeter $8p$ and $m\angle IEQ = 55°$

37. Kite *KITE* with $TE = 2EK$ and $m\angle TEK = 120°$

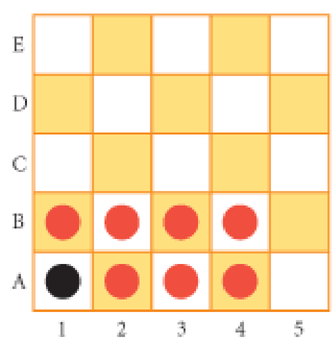
IMPROVING YOUR REASONING SKILLS

Checkerboard Puzzle

1. Four checkers—three red and one black—are arranged on the corner of a checkerboard, as shown at right. Any checker can jump any other checker. The checker that was jumped over is then removed. With exactly three horizontal or vertical jumps, remove all three red checkers, leaving the single black checker. Record your solution.



2. Now, with exactly seven horizontal or vertical jumps, remove all seven red checkers, leaving the single black checker. Record your solution.



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LESSON

1.8

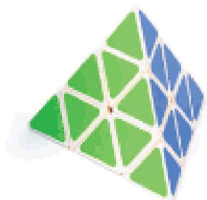
Space Geometry

When curiosity turns to serious matters, it's called research.

MARIE VON EBNER-ESCHENBACH

Lesson 1.1 introduced you to point, line, and plane. Throughout this chapter you have used these terms to define a wide range of other geometric figures, from rays to polygons. You did most of your work on a single flat surface, a single plane. Some problems, however, required you to step out of a single plane to visualize geometry in space. In this lesson you will learn more about space geometry, or solid geometry.

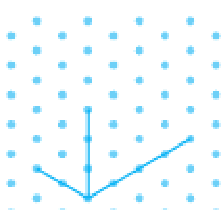
Space is the set of all points. Unlike one-dimensional lines and two-dimensional planes, space cannot be contained in a flat surface. Space is three-dimensional, or "3-D."



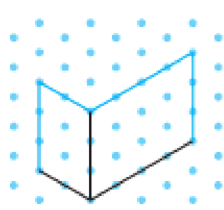
In an "edge view," you see the front edge of a building as a vertical line, and the other edges as diagonal lines. Isometric dot paper helps you draw these lines, as you can see in the steps below.

Let's practice the visual thinking skill of presenting three-dimensional (3-D) objects in two-dimensional (2-D) drawings.

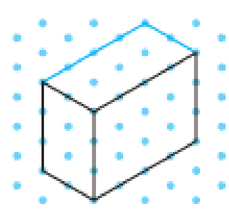
The geometric solid you are probably most familiar with is a box, or rectangular prism. Below are steps for making a two-dimensional drawing of a rectangular prism. This type of drawing is called an **isometric drawing.** It shows three sides of an object in one view (an edge view). This method works best with isometric dot paper. After practicing, you will be able to draw the box without the aid of the dot grid.



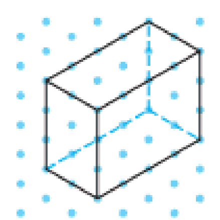
Step 1



Step 2



Step 3



Step 4

Use dashed lines for edges that you couldn't see if the object were solid.

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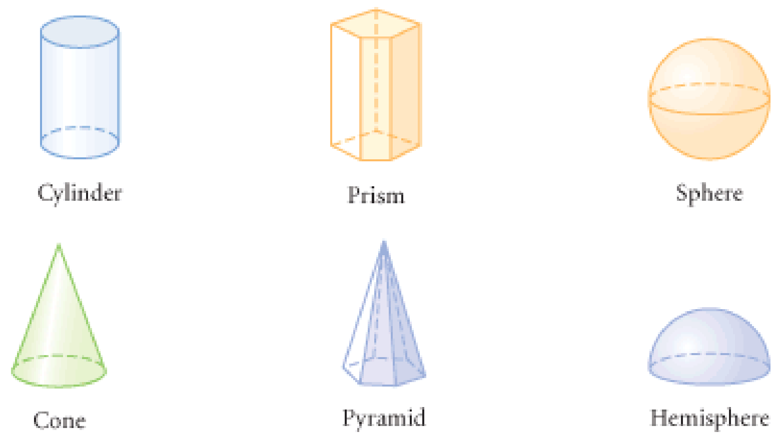
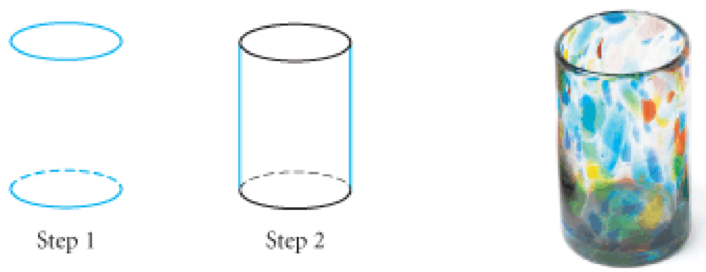
The three-dimensional objects you will study include the six types of geometric solids shown below.

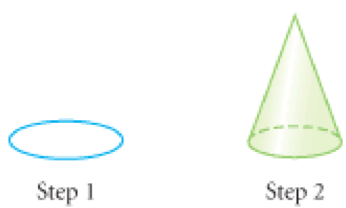


The shapes of these solids are probably already familiar to you even if you are not familiar with their proper names. The ability to draw these geometric solids is an important visual thinking skill. Here are some drawing tips. Remember to use dashes for the hidden lines.

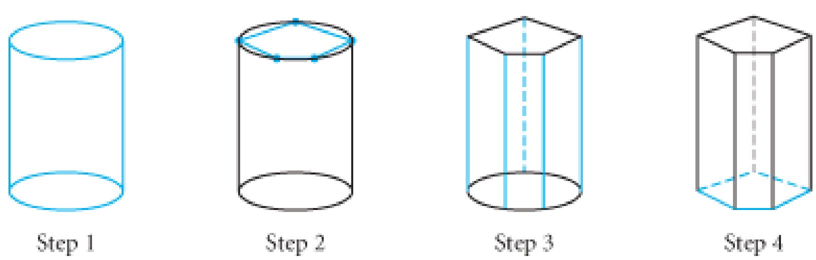
Cylinder



Cone



Prism



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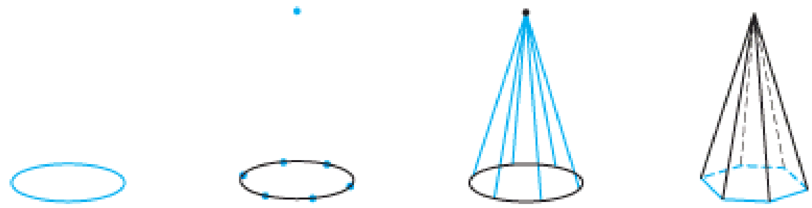
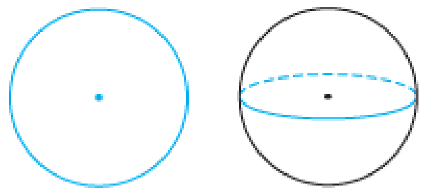
Pyramid



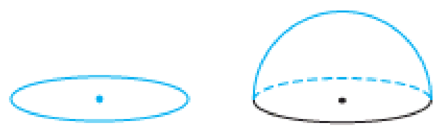
Step 1 Step 2 Step 3 Step 4

Sphere



Step 1 Step 2

Hemisphere



Step 1 Step 2

Solid geometry also involves visualizing points and lines in space. In the following investigation, you will have to visualize relationships between geometric figures in a plane and in space.

Investigation
Space Geometry

Step 1 Make a sketch or use physical objects to demonstrate each statement in the list below.

Step 2 Work with your group to determine whether each statement is true or false. If the statement is false, draw a picture and explain why it is false.

1. For any two points, there is exactly one line that can be drawn through them.
2. For any line and a point not on the line, there is exactly one plane that can contain them.
3. For any two lines, there is exactly one plane that contains them.
4. If two coplanar lines are both perpendicular to a third line in the same plane, then the two lines are parallel.
5. If two planes do not intersect, then they are parallel.
6. If two lines do not intersect, then they are parallel.
7. If a line is perpendicular to two lines in a plane, and the line is not contained in the plane, then the line is perpendicular to the plane.

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EXERCISES

For Exercises 1–6, draw each figure. Study the drawing tips provided on the previous page before you start.

1. Cylinder

2. Cone

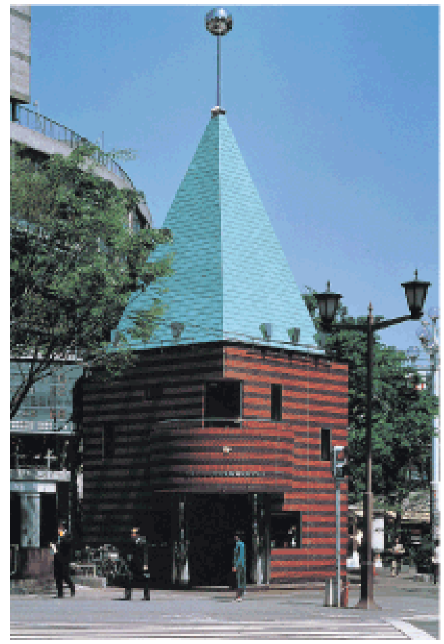
3. Prism with a hexagonal base

4. Sphere

5. Pyramid with a heptagonal base

6. Hemisphere

7. The photo at right shows a prism-shaped building with a pyramid roof and a cylindrical porch. Draw a cylindrical building with a cone roof and a prism-shaped porch.



A police station, or *koban,* in Tokyo, Japan

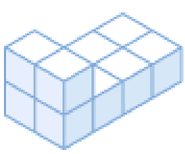
For Exercises 8 and 9, make a drawing to scale of each figure. Use isometric dot paper. Label each figure. (For example, in Exercise 8, draw the solid so that the dimensions measure 2 units by 3 units by 4 units, then label the figure with meters.)

8. A rectangular solid 2 m by 3 m by 4 m, sitting on its biggest face.

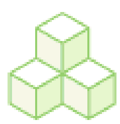
9. A rectangular solid 3 inches by 4 inches by 5 inches, resting on its smallest face. Draw lines on the three visible surfaces showing how you can divide the solid into cubic-inch boxes. How many such boxes will fit in the solid?

For Exercises 10–12, use isometric dot paper to draw the figure shown.

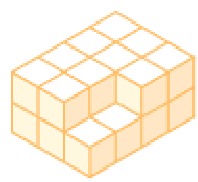
10.



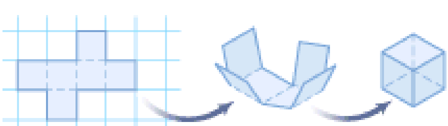
11.



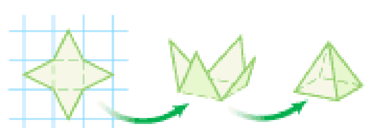
12.



A **net** is a two-dimensional pattern that you can cut and fold to form a three-dimensional figure. Another visual thinking skill you will need is the ability to visualize nets being folded into solid objects and geometric solids being unfolded into nets. The net below left can be folded into a cube and the net below right can be folded into a pyramid.



Net for a cube



Net for a square-based pyramid

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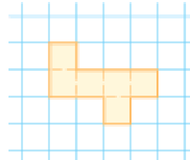
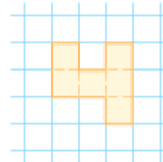
13. Which net(s) will fold to make a cube?

A.

B.



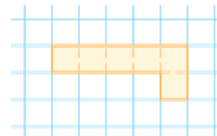
C.



D.

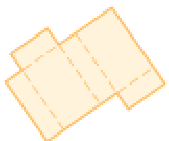
E.

F.

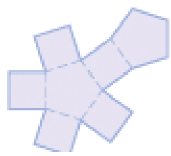


For Exercises 14–17, match the net with its geometric solid.

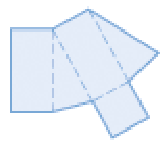
14.



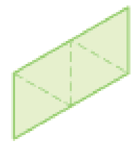
15.



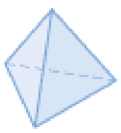
16.



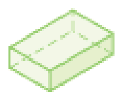
17.



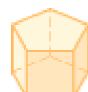
A.



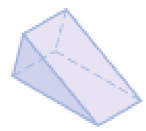
B.



C.

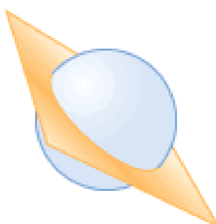


D.

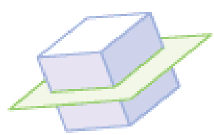


When a solid is cut by a plane, the resulting two-dimensional figure is called a **section.** For Exercises 18 and 19, sketch the section formed when each solid is sliced by the plane, as shown.

18.



19.



Slicing a block of clay reveals a section of the solid. Here, the section is a rectangle.

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All of the statements in Exercises 20–27 are true except for two. Make a sketch to demonstrate each true statement. For each false statement, draw a sketch and explain why it is false.

20. Only one plane can pass through three noncollinear points.

21. If a line intersects a plane that does not contain the line, then the intersection is exactly one point.

22. If two lines are perpendicular to the same line, then they are parallel. ⓗ

23. If two different planes intersect, then their intersection is a line.

Physical models can help you visualize the intersections of lines and planes in space. Can you see examples of intersecting lines in this photo? Parallel lines? Planes? Points?

24. If a line and a plane have no points in common, then they are parallel.

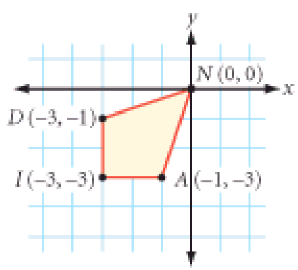
25. If a plane intersects two parallel planes, then the lines of intersection are parallel.

26. If three planes intersect, then they divide space into six parts.

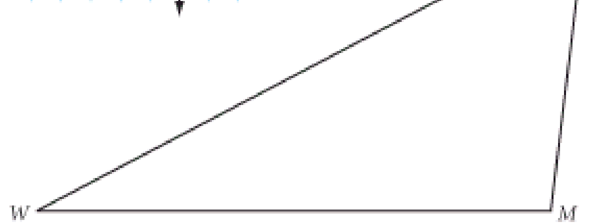
27. If two lines are perpendicular to the same plane, then they are parallel to each other.

Review

28. If the kite *DIAN* were rotated 90° clockwise about the origin, to what location would point *A* be relocated?



29. Use your ruler to measure the perimeter of $\triangle WIM$ (in centimeters) and your protractor to measure the largest angle.



30. Use your geometry tools to draw a triangle with two sides of length 8 cm and length 13 cm and the angle between them measuring 120°.

IMPROVING YOUR VISUAL THINKING SKILLS

Equal Distances

Here's a real challenge:

Show four points *A, B, C,* and *D* so that $AB = BC = AC = AD = BD = CD$.

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LESSON

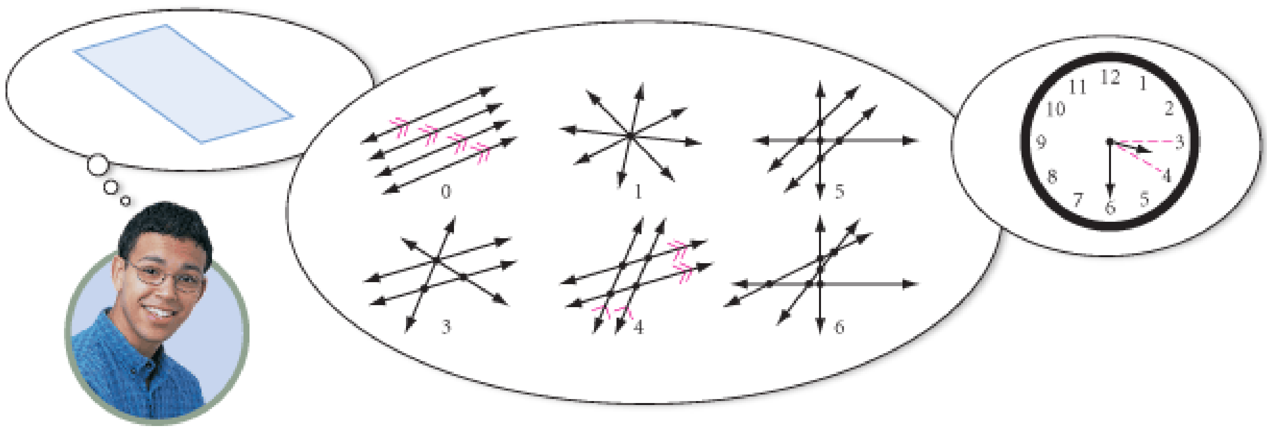
1.9

A Picture Is Worth a Thousand Words

You can observe a lot just by watching.
YOGI BERRA

A picture is worth a thousand words! That expression certainly applies to geometry. A drawing of an object often conveys information more quickly than a long written description. People in many occupations use drawings and sketches to communicate ideas. Architects create blueprints. Composers create musical scores. Choreographers visualize and map out sequences of dance steps. Basketball coaches design plays. Interior designers—well, you get the picture.

Visualization skills are extremely important in geometry. So far, you have visualized geometric situations in every lesson. To visualize a plane, you pictured a flat surface extending infinitely. In another lesson you visualized the number of different ways that four lines can intersect. Can you picture what the hands of a clock look like when it is 3:30?

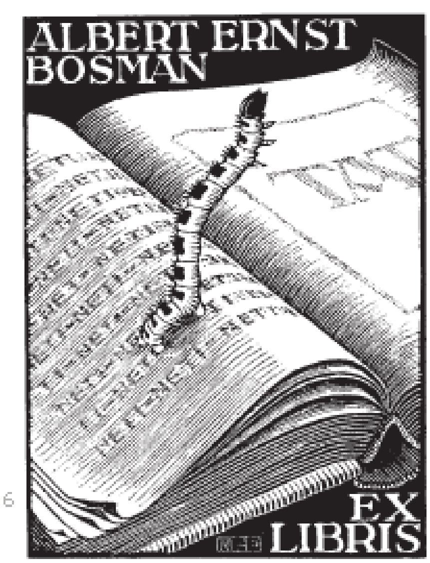


By drawing diagrams, you apply visual thinking to problem solving. Let's look at some examples that show how to use visual thinking to solve word problems.

EXAMPLE A

Volumes 1 and 2 of a two-volume set of math books sit next to each other on a shelf. They sit in their proper order: Volume 1 on the left and Volume 2 on the right. Each front and back cover is $\frac{1}{8}$-inch thick, and the pages portion of each book is 1-inch thick. If a bookworm starts at the first page of Volume 1 and burrows all the way through to the last page of Volume 2, how far will it travel?

Take a moment and try to solve the problem in your head.



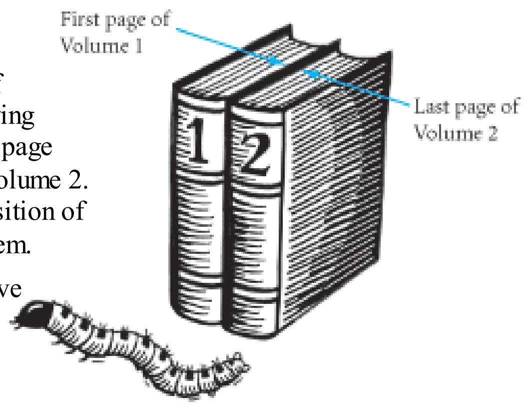
Bookplate for Albert Ernst Bosman, M. C. Escher, 1946
©2002 Cordon Art B. V.–Baarn–Holland. All rights reserved.

Solution

Did you get $2\frac{1}{4}$ inches? It seems reasonable, doesn't it?

However, that's not the answer. Let's reread the problem to identify what information is given and what we are asked to find.

We are given the thickness of each cover, the thickness of the pages portion, and the position of the books on the shelf. We are trying to find how far it is from the first page of Volume 1 to the last page of Volume 2. Draw a picture and locate the position of the pages referred to in the problem.



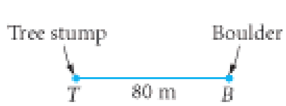
Now "look" how easy it is to solve the problem. The bookworm traveled only $\frac{1}{4}$ inch through the two covers!

EXAMPLE B

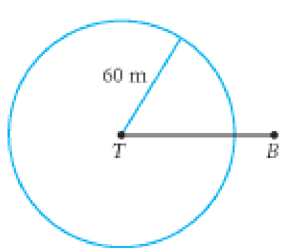
Harold, Dina, and Linda are standing on a flat, dry field reading their treasure map. Harold is standing at one of the features marked on the map, a gnarled tree stump, and Dina is standing atop a large black boulder. The map shows that the treasure is buried 60 meters from the tree stump and 40 meters from the large black boulder. Harold and Dina are standing 80 meters apart. What is the locus of points where the treasure might be buried?

Solution

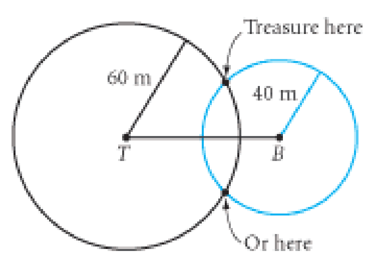
Start by drawing a diagram based on the information given in the first two sentences, then add to the diagram as new information is added. Can you visualize all the points that are 60 meters from the tree stump? Mark them on your diagram. They should lie on a circle. The treasure is also 40 meters from the boulder. All the possible points lie in a circle around the boulder. The two possible spots where the treasure might be buried are the points where the two circles intersect.



Initial diagram



Improved diagram



Final diagram

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As in the previous example, when there is more than one point or even many points that satisfy a set of conditions, the set of points is called a **locus.**

keymath.com/DG

You can extend the scenario from Example B to explore different types of solutions for similar l problems in the **Dynamic Geometry Exploration** Treasure Hunt at www.keymath.com/DG .

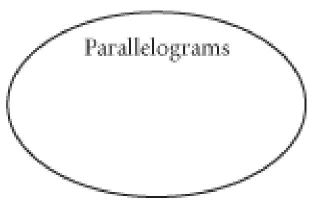
A diagram can also help organize information to help make sense of difficult concepts. A **Venn diagram** represents larger groups that contain smaller groups as circles within circles, or ovals within ovals. For example, a larger circle for “high school students” would contain a smaller circle for “sophomores.” Overlapping circles show that it is possible to belong to two different groups at the same time, such as “sophomores” and “geometry students.” Let’s look at an example, using some of the quadrilateral definitions you wrote in Lesson 1.6.

EXAMPLE C

Create a Venn diagram to show the relationships among parallelograms, rhombuses, rectangles, and squares.

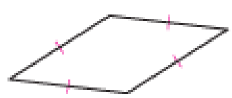
Solution

Start by deciding what is the most general group. What do parallelograms, rhombuses, rectangles, and squares have in common? They all have two pairs of parallel sides, so parallelograms is the largest oval.



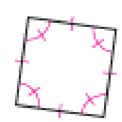
Now consider the special characteristics of rhombuses, rectangles, and squares.

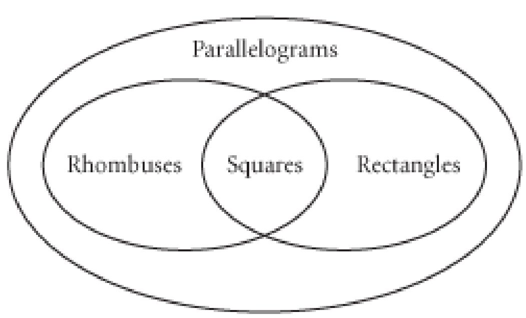
Rhombuses have four congruent sides, so they are equilateral.



Rectangles have four congruent angles, so they are equiangular.

Squares are both equilateral and equiangular. They have the characteristics of rhombuses and rectangles, so they belong to both groups. This can be shown by using overlapping ovals.





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EXERCISES

You will need

Construction tools for Exercises 35 and 36

1. Surgeons, engineers, carpenters, plumbers, electricians, and furniture movers all rely on trained experience with visual thinking. Describe how one of these tradespeople or someone in another occupation uses visual thinking in his or her work.

Read each problem, determine what you are trying to find, draw a diagram, and solve the problem.

2. In the city of Rectangulus, all the streets running east–west are numbered and those streets running north–south are lettered. The even-numbered streets are one-way east and the odd-numbered streets are one-way west. All the vowel-lettered avenues are one-way north and the rest are two-way. Can a car traveling south on S Street make a legal left turn onto 14th Street?

3. Midway through a 2000-meter race, a photo is taken of five runners. It shows Meg 20 meters behind Edith. Edith is 50 meters ahead of Wanda, who is 20 meters behind Olivia. Olivia is 40 meters behind Nadine. Who is ahead? In your diagram, use M for Meg, E for Edith, and so on.

4. Mary Ann is building a fence around the outer edge of a rectangular garden plot that measures 25 feet by 45 feet. She will set the posts 5 feet apart. How many posts will she need?

5. Freddie the Frog is at the bottom of a 30-foot well. Each day he jumps up 3 feet, but then, during the night, he slides back down 2 feet. How many days will it take Freddie to get to the top and out? (h)

6. Here is an exercise taken from Marilyn vos Savant's Ask Marilyn® column in *Parade* magazine. It is a good example of a difficult-sounding problem becoming clear once a diagram has been made. Try it. (h)

   A 30-foot cable is suspended between the tops of two 20-foot poles on level ground. The lowest point of the cable is 5 feet above the ground. What is the distance between the two poles?

7. Points A and B lie in a plane. Sketch the locus of points in *the plane* that are equally distant from points A and B. Sketch the locus of points in *space* that are equally distant from points A and B. (h)

8. Draw an angle. Label it $\angle A$. Sketch the locus of points in the plane of angle A that are the same distance from the two sides of angle A.

9. Line AB lies in plane $\mathscr{P}$. Sketch the locus of points in plane $\mathscr{P}$ that are 3 cm from $\overleftrightarrow{AB}$. Sketch the locus of points in space that are 3 cm from $\overleftrightarrow{AB}$.

10. Create a Venn diagram showing the relationships among triangles, trapezoids, polygons, obtuse triangles, quadrilaterals, and isosceles triangles.

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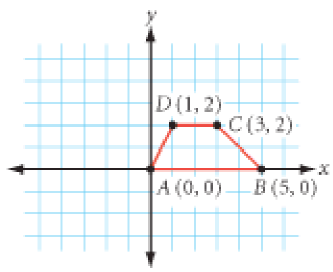
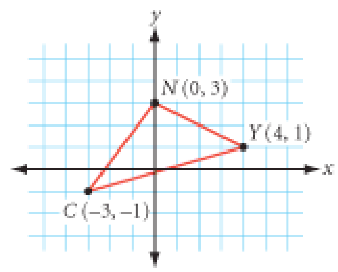
11. Beth Mack and her dog Trouble are exploring in the woods east of Birnam Woods Road, which runs north-south. They begin walking in a zigzag pattern: 1 km south, 1 km west, 1 km south, 2 km west, 1 km south, 3 km west, and so on. They walk at the rate of 4 km/h. If they started 15 km east of Birnam Woods Road at 3:00 P.M., and the sun sets at 7:30 P.M., will they reach Birnam Woods Road before sunset?

In geometry you will use visual thinking all the time. In Exercises 12 and 13 you will be asked to locate and recognize congruent geometric figures even if they are in different positions due to translations (slides), rotations (turns), or reflections (flips).

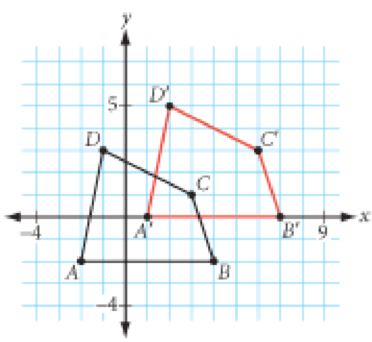
12. If trapezoid *ABCD* were rotated 90° counterclockwise about (0, 0), to what (x, y) location would points *A, B, C,* and *D* be relocated?



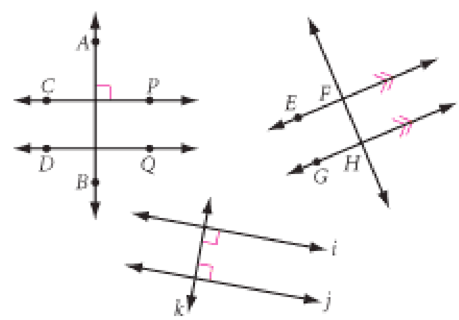
13. If $\triangle CYN$ were reflected across the y-axis, to what location would points *C, N,* and *Y* be relocated?



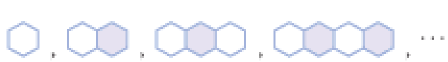
14. What was the ordered pair rule used to relocate the four vertices of *ABCD* to *A′B′C′D′*?



15. Which lines are perpendicular? Which lines are parallel?



16. Sketch the next two figures in the pattern below. If this pattern were to continue, what would be the perimeter of the eighth figure in the pattern? (Assume the length of each segment is 1 cm.)



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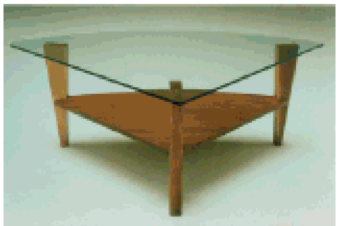
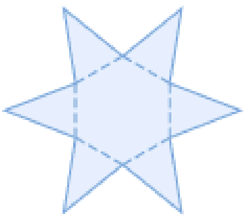
17. A tabletop represents a plane. Examine the combination of points and lines that hold each tabletop in place. Removing one point or line would cause the tabletop to wobble or fall. In geometry, we say that these combinations of points and lines **determine** a plane. For each photo, use geometric terms to describe what determines the plane represented by the tabletop.



For Exercises 18–20, sketch the three-dimensional figure formed by folding each net into a solid. Name the solid.

18.



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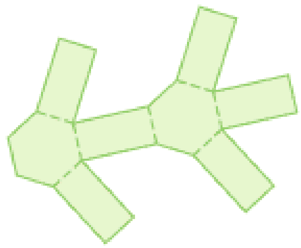

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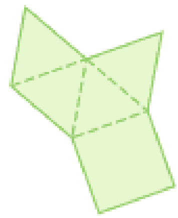
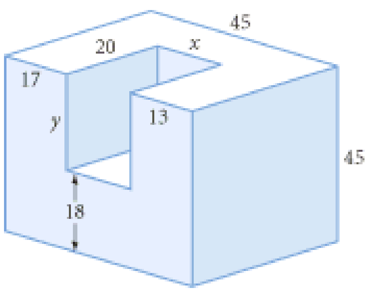
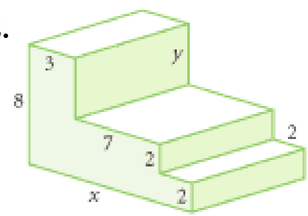


For Exercises 21 and 22, find the lengths *x* and *y*. (Every angle on each block is a right angle.)

21.



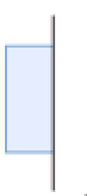
22.



A real-life example of a "solid of revolution" is a clay pot on a potter's wheel.

In Exercises 23 and 24, each figure represents a two-dimensional figure with a wire attached. The three-dimensional solid formed by spinning the figure on the wire between your fingers is called a **solid of revolution.** Sketch the solid of revolution formed by each two-dimensional figure.

23.



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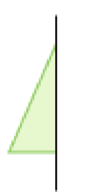

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Review

For Exercises 25–34, write the words or the symbols that make the statement true.

25. The three undefined terms of geometry are _?_, _?_, and _?_.

26. "Line *AB*" may be written using a symbol as _?_.

27. "Arc *AB*" may be written using a symbol as _?_.

28. The point where the two sides of an angle meet is the _?_ of the angle.

29. "Ray *AB*" may be written using a symbol as _?_.

30. "Line *AB* is parallel to segment *CD*" is written in symboli form as _?_.

31. The geometry tool you use to measure an angle is a _?_.

32. "Angle *ABC*" is written in symbolic form as _?_.

33. The sentence "Segment *AB* is perpendicular to line *CD*" is written in symbolic form as _?_.

34. The angle formed by a light ray coming into a mirror is _?_ the angle formed by a light ray leaving the mirror.

William Thomas Williams, DO YOU THINK A IS B, acrylic on canvas, 1975–77, Fisk University Galleries, Nashville, Tennessee.

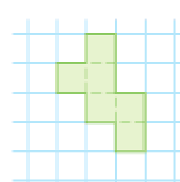
35. Use your compass and straightedge to draw two congruent circles intersecting in exactly one point. How does the distance between the two centers compare with the radius?

36. Use your compass and straightedge to construct two congruent circles so that each circle passes through the center of the other circle. Label the centers *P* and *Q*. Construct $\overleftrightarrow{PQ}$ connecting the centers. Label the points of intersection of the two circles *A* and *B*. Construct chord *AB*. What is the relationship between $\overline{AB}$ and $\overleftrightarrow{PQ}$?

IMPROVING YOUR VISUAL THINKING SKILLS

Hexominoes

Polyominoes with six squares are called hexominoes. There are 35 different hexominoes. There is 1 with a longest string of six squares; there are 3 with a longest string of five squares, and 1 with a longest string of two squares. The rest have a longest string of either four squares or three squares. Use graph paper to sketch all 35 hexominoes. Which ones are nets for cubes? Here is one hexomino that does fold into a cube.



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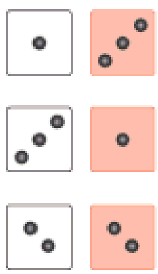
Exploration

Geometric Probability I



You probably know what probability means. The **probability**, or likelihood, of a particular outcome is the ratio of the number of successful outcomes to the number of possible outcomes. So the probability of rolling a 4 on a 6-sided die is $\frac{1}{6}$. Or you can name an event that involves more than one outcome, like getting the total 4 on two 6-sided dice. Because each die can come up in six different ways, there are 6×6, or 36, combinations (count 'em!). You can get the total 4 with a 1 and a 3, a 3 and a 1, or a 2 and a 2. So the probability of getting the total 4 is $\frac{3}{36}$, or $\frac{1}{12}$. Anyway, that's the theory.

Activity

Chances Are

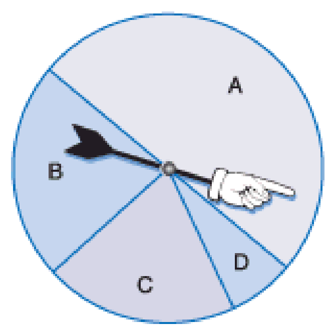
You will need

- a protractor
- a ruler

In this activity you'll see that you can apply probability theory to geometric figures.

The Spinner

After you've finished your homework and have eaten dinner, you play a game of chance using the spinner at right. Where the spinner lands determines how you'll spend the evening.



Sector A: Playing with your younger brother the whole evening

Sector B: Half the evening playing with your younger brother and half the evening watching TV

Sector C: Cleaning the birdcage, the hamster cage, and the aquarium the whole evening

Sector D: Playing in a band in a friend's garage the whole evening

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Step 1 What is the probability of landing in each sector?

Step 2 What is the probability that you'll spend at least half the evening with your younger brother? What is the probability that you won't spend any time with him?

The Bridge

A computer programmer who is trying to win money on a TV survival program builds a 120-ft rope bridge across a piranha-infested river 90 ft below.

Step 3 If the rope breaks where he is standing (a random point), but he is able to cling to one end of it, what is the probability that he'll avoid getting wet (or worse)?

Step 4 Suppose the probability that the rope breaks at all is $\frac{1}{2}$. Also suppose that, as long as he doesn't fall more than 30 ft, the probability that he can climb back up is $\frac{3}{4}$. What is the probability that he won't fall at all? What is the probability that if he does, he'll be able to climb back up?

The Bus Stop

Noriko arrives at the bus stop at a random time between 3:00 and 4:30 P.M. each day. Her bus stops there every 20 minutes, including at 3:00 P.M.

Step 5 Draw a number line to show stopping times. (Don't worry about the length of time that the bus is actually stopped. Assume it is 0 minutes.)

Step 6 What is the probability that she will have to wait 5 minutes or more? 10 minutes or more? Hint: What line lengths represent possible waiting time?

Step 7 If the bus stops for exactly 3 minutes, how do your answers to Step 6 change?

Step 8 List the geometric properties you needed in each of the three scenarios above and tell how your answers depended on them.

Step 9 How is geometric probability like the probability you've studied before? How is it different?

Step 10 Create your own geometric probability problem.

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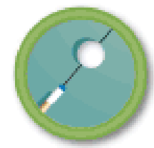
CHAPTER 1 REVIEW

It may seem that there's a lot to memorize in this chapter. But having defined terms yourself, you're more likely to remember and understand them. The key is to practice using these new terms and to be organized. Do the following exercises, then read Assessing What You've Learned for tips on staying organized.

Whether you've been keeping a good list or not, go back now through each lesson in the chapter and double-check that you've completed each definition and that you understand it. For example, if someone mentions a geometry term to you, can you sketch it? If you are shown a geometric figure, can you name it? Compare your list of geometry terms with the lists of your group members.



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EXERCISES

Answers to all exercises in every Chapter Review are provided in the back of the book.

For Exercises 1–16, identify the statement as true or false. For each false statement, explain why it is false or sketch a counterexample.

1. The three basic building blocks of geometry are point, line, and plane.
2. "The ray through point P from point Q" is written in symbolic form as $\overrightarrow{PQ}$.
3. "The length of segment PQ" can be written as PQ.
4. The vertex of angle PDQ is point P.
5. The symbol for *perpendicular* is $\perp$.
6. A scalene triangle is a triangle with no two sides the same length.
7. An acute angle is an angle whose measure is more than 90°.
8. If $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point P, then $\angle APD$ and $\angle APC$ are a pair of vertical angles.
9. A diagonal is a line segment in a polygon connecting any two nonconsecutive vertices.
10. If two lines lie in the same plane and are perpendicular to the same line, then they are parallel.
11. If the sum of the measures of two angles is 180°, then the two angles are complementary.
12. A trapezoid is a quadrilateral having exactly one pair of parallel sides.
13. A polygon with ten sides is a decagon.

A knowledge of parallel lines, planes, arcs, circles, and symmetry is necessary to build durable guitars that sound pleasing.

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14. A square is a rectangle with all the sides equal in length.

15. A pentagon has five sides and six diagonals.

16. The largest chord of a circle is a diameter of the circle.

For Exercises 17–25, match each term with its figure below, or write "no match."

17. Octagon

18. Isosceles right triangle

19. Rhombus

20. Trapezoid

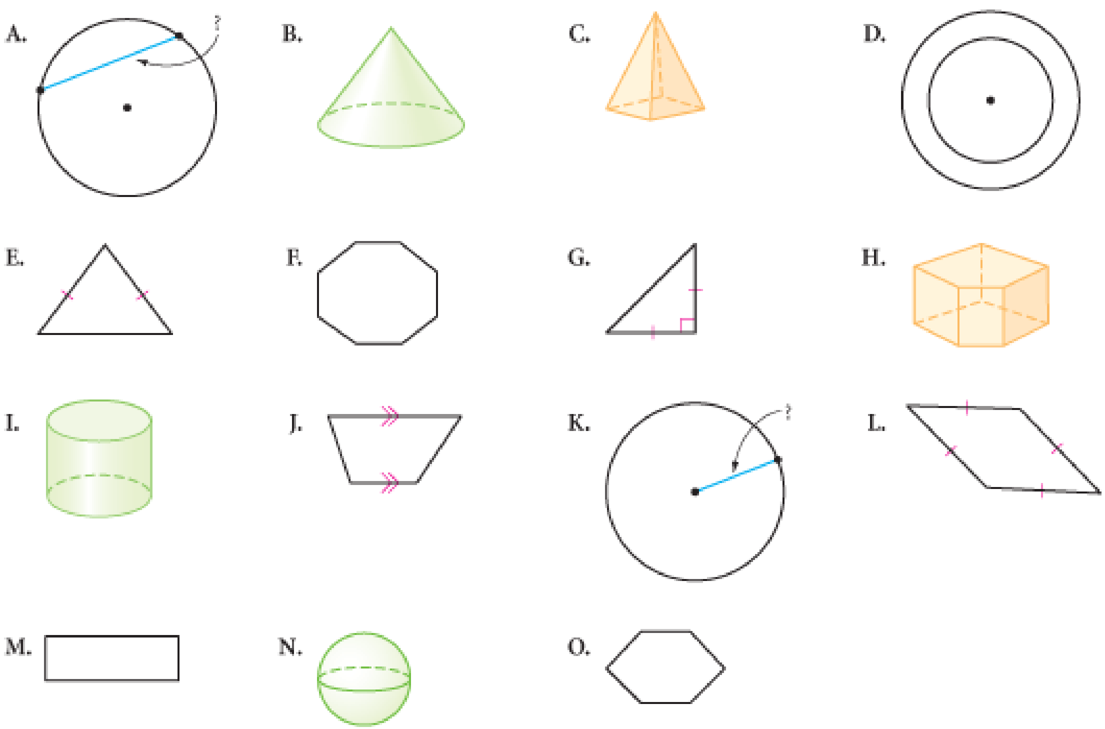
21. Pyramid

22. Cylinder

23. Concave polygon

24. Chord

25. Minor arc



For Exercises 26–33, sketch, label, and mark each figure.

26. Kite *KYTE* with $\overline{KY} \cong \overline{YT}$

27. Scalene triangle *PTS* with *PS* = 3, *ST* = 5, *PT* = 7, and angle bisector $\overline{SO}$

28. Hexagon *REGINA* with diagonal $\overline{AG}$ parallel to sides $\overline{RE}$ and $\overline{NI}$

29. Trapezoid *TRAP* with $\overline{AR}$ and $\overline{PT}$ the nonparallel sides. Let *E* be the midpoint of $\overline{PT}$ and let *Y* be the midpoint of $\overline{AR}$ Draw $\overline{EY}$.

30. A triangle with exactly one line of reflectional symmetry

31. A circle with center at *P,* radii $\overline{PA}$ and $\overline{PT}$, and chord $\overline{TA}$ creating a minor arc $\overset{\frown}{TA}$

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32. A pair of concentric circles with the diameter $\overline{AB}$ of the inner circle perpendicular at B to a chord $\overline{CD}$ of the larger circle

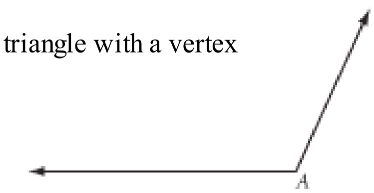
33. A pyramid with a pentagonal base

34. Draw a rectangular prism 2 inches by 3 inches by 5 inches, resting on its largest face. Draw lines on the three visible faces, showing how the solid can be divided into 30 smaller cubes.

35. Use your protractor to draw a 125° angle.

36. Use your protractor, ruler, and compass to draw an isosceles triangle with a vertex angle having a measure of 40°.

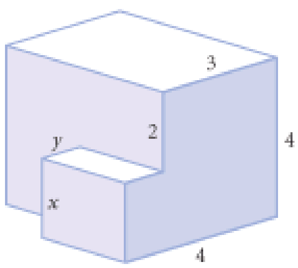
37. Use your geometry tools to draw a regular octagon.



38. What is the measure of $\angle A$? Use your protractor.

For Exercises 39–42, find the lengths x and y. (Every angle on each block is a right angle.)

39.



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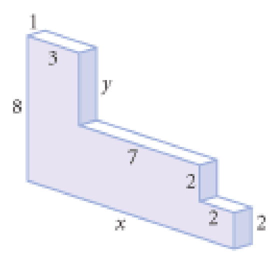

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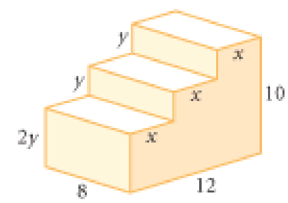
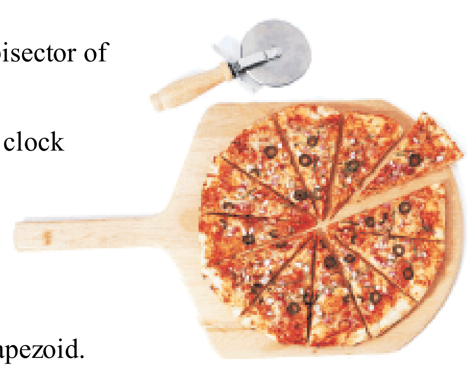


42.

43. If D is the midpoint of $\overline{AC}$, is the midpoint of $\overline{AB}$, and BD = 12 cm, what is the length of $\overline{AB}$?

44. If $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$ and $\overrightarrow{BE}$ is the angle bisector of $\angle DBC$, find $m\angle EBA$ if $m\angle DBE = 32°$.



45. What is the measure of the angle formed by the hands of the clock at 2:30?

46. If the pizza is cut into 12 congruent pieces, how many degrees are in each central angle?

47. Make a Venn diagram to show the relationships among these shapes: quadrilaterals, rhombus, rectangle, square, trapezoid.

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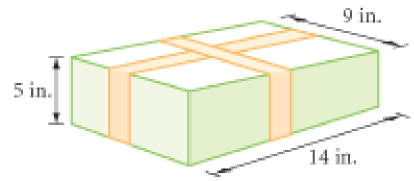
48. The box at right is wrapped with two strips of ribbon, as shown. What is the minimum length of ribbon needed to decorate the box?

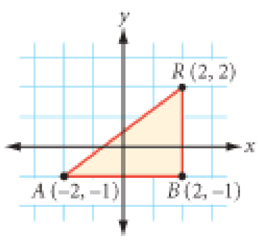


49. At one point in a race, Rico was 15 ft behind Paul and 18 ft ahead of Joe. Joe was trailing George by 30 ft. Paul was ahead of George by how many ft?

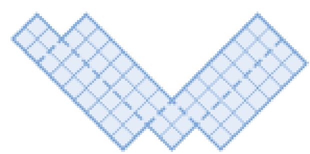
50. A large aluminum ladder was resting vertically against the research shed at midnight, when it began to slide down the side of the shed. A burglar was clinging to the ladder's midpoint, holding a pencil flashlight that was visible in the dark. Witness Jill Seymour claimed to see the ladder slide. What did she see? That is, what was the path taken by the bulb of the flashlight? Draw a diagram showing the path. (Devise a physical test to check your visual thinking. You might try sliding a meterstick against a wall, or you might plot points on graph paper.)

51. Jiminey Cricket is caught in a windstorm. At 5:00 P.M. he is 500 cm away from his home. Each time he jumps toward home, he leaps a distance of 50 cm, but before he regains strength to jump again, he is blown back 40 cm. If it takes a full minute between jumps, what time will Jiminey get home?

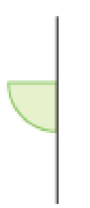
52. If the right triangle *BAR* were rotated 90° clockwise about point *B*, to what location would point *A* be relocated?



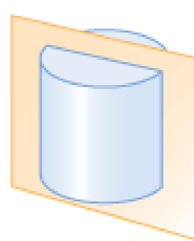
53. Sketch the three-dimensional figure formed by folding the net below into a solid.



54. Sketch the solid of revolution formed when you spin the two-dimensional figure about the line.



55. Sketch the section formed when the solid is sliced by the plane, as shown.



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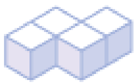
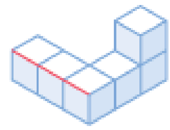
56. Use an isometric dot grid to sketch the figure shown below.

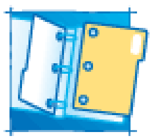


57. Sketch the figure shown with the red edge vertical and facing the viewer.



Assessing What You've Learned

ORGANIZE YOUR NOTEBOOK



Is this textbook filling up with folded-up papers stuffed between pages? If so, that's a bad sign! But it's not too late to get organized. Keeping a well-organized notebook is one of the best habits you can develop to improve and assess your learning. You should have sections for your classwork, definition list, and homework exercises. There should be room to make corrections, to summarize what you learned, and to write down questions you still have.

Many books include a definition list (sometimes called a glossary) in the back. This book makes you responsible for your own glossary, so it's essential that, in addition to taking good notes, you keep a complete definition list that you can refer to. You started a definition list in Lesson 1.1. Get help from classmates or your teacher on any definition you don't understand.

As you progress through the course, your notebook will become more and more important. A good way to review a chapter is to read through the chapter and your notes and write a one-page summary of the chapter. If you create a one-page summary for each chapter, the summaries will be very helpful to you when it comes time for midterms and final exams. You'll find no better learning and study aid than a summary page for each chapter, and your definition list, kept in an organized notebook.

UPDATE YOUR PORTFOLIO

- If you did the project in this chapter, document your work and add it to your portfolio.
- Choose one homework assignment that demonstrates your best work in terms of completeness, correctness, and neatness. Add it (or a copy of it) to your portfolio.

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